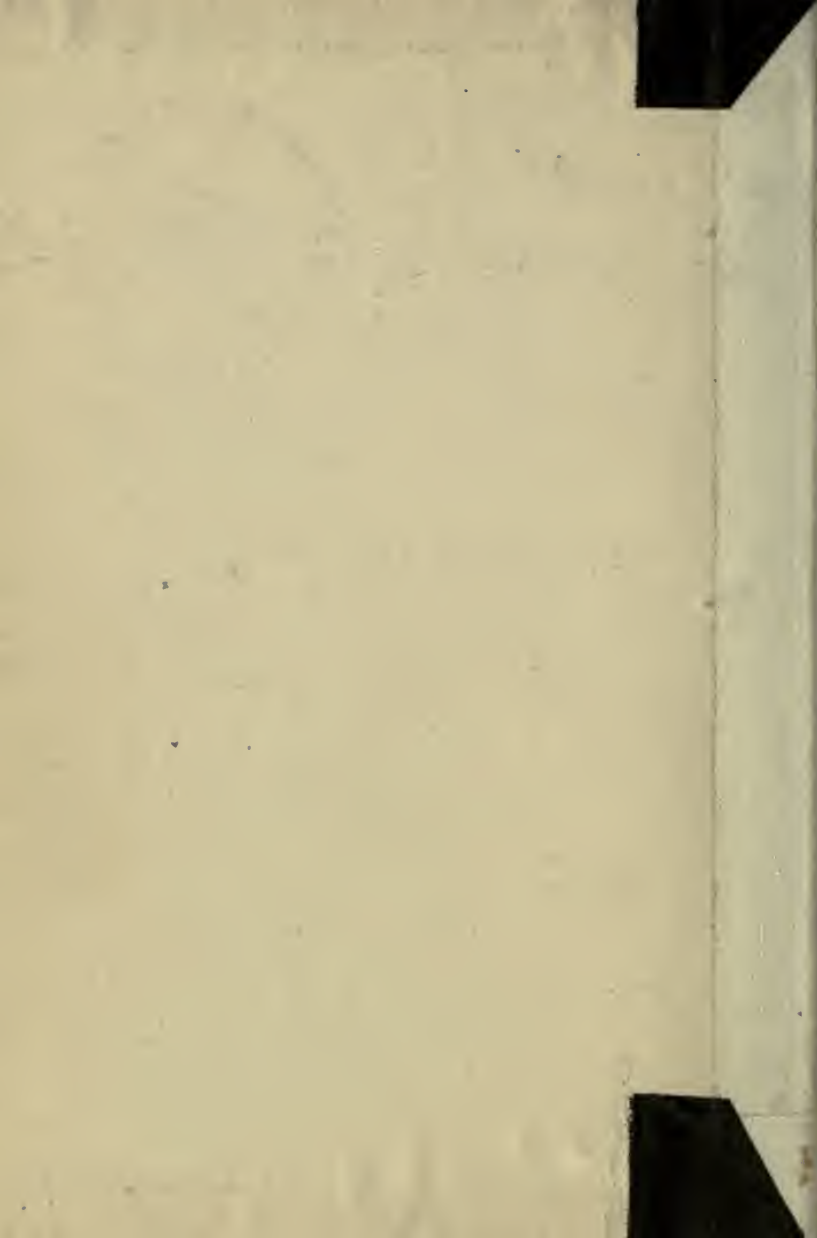
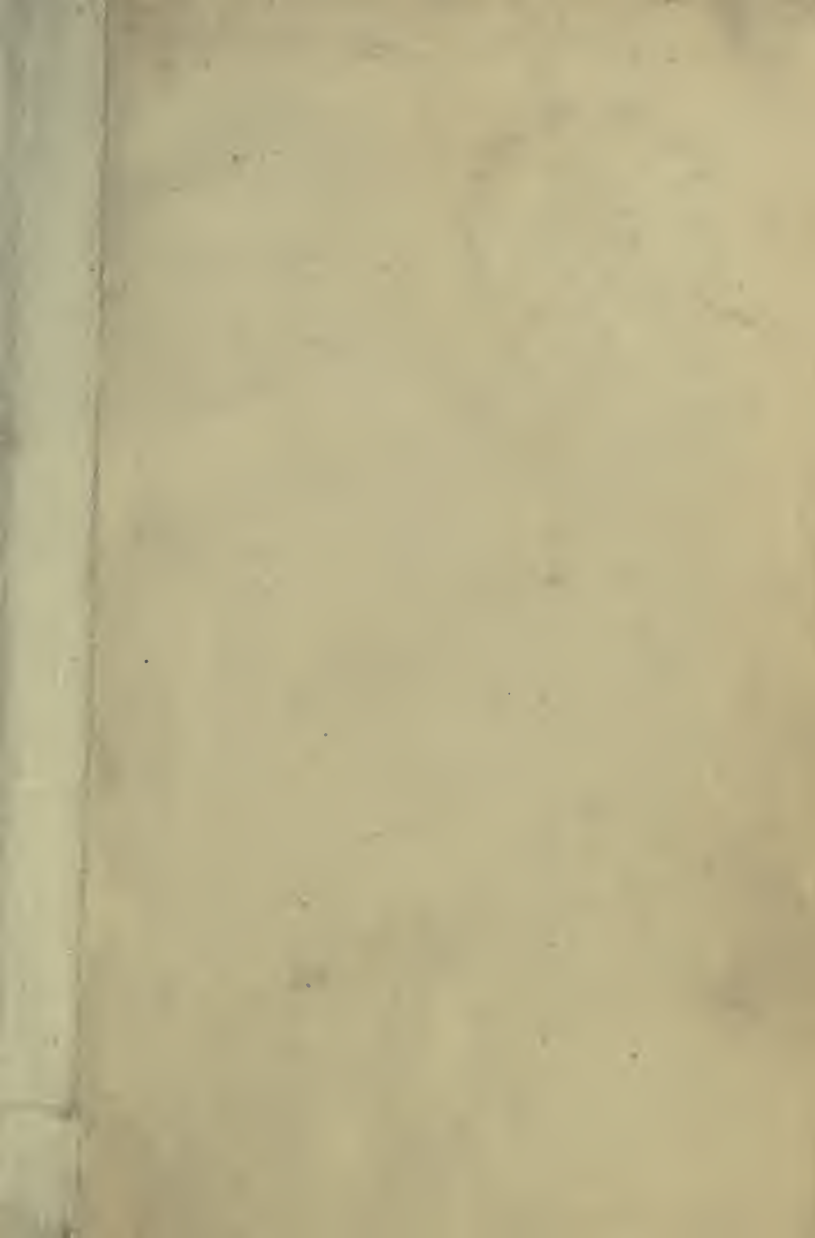


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AN
ELEMENTARY TEXT-BOOK
OF
HEAT AND LIGHT.

WITH ONE HUNDRED AND FIFTY ILLUSTRATIONS,

AND NUMEROUS EXAMPLES.

BY
R. WALLACE STEWART, B.Sc. LOND.

FIRST IN FIRST CLASS HONOURS IN PHYSICS.



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P R E F A C E .

THIS work is intended to serve as an introduction to the study of Heat and Light, and fully covers the ground defined by the Syllabus for the London University Matriculation Examination. Although primarily written for the use of candidates for this examination, the book is intended to be suitable for general use. In order to avoid the confusion which always arises from a too close observance of the requirements of different examinations, I have considered the Matriculation Syllabus as an indication of what our best authorities consider to be a suitable elementary course, and have treated the subject accordingly. My chief aim has been to give the student clear and accurate notions of the elementary principles of the subject, and to enable him to obtain a firm grasp of these principles by practical illustration and by exercises on their application to problems.

For general convenience the reference numbers in this book are arranged to correspond with those in our more advanced Text-books of Heat and Light to which the reader desirous of extending his study of these subjects is referred.

I have to acknowledge suggestions from the Principal of the University Correspondence College, and from Mr. S. Andrews, B.A., B.Sc., who has kindly read through all the proofs.

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R. W. S.

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HEAT.

CHAPTER I.

INTRODUCTORY.

I. Preliminary considerations. The sensations of heat and cold are familiar to all. When we touch any substance with the hand, we are at once able to say whether it feels hot or cold; but it is evident that by this we mean that the substance is hotter or colder than the hand. We can convince ourselves of this by placing, for a short time, one hand in warm water and the other in cold, and then plunging both into lukewarm water, which will feel hot to one hand and cold to the other. These sensations of heat and cold are therefore relative, and tell us nothing of the actual state of the body producing the sensation.

On the other hand, we know from these sensations that different thermal states of any given body exist. For example, if both hands be immersed for some time in lukewarm water, and then placed, one in hot water and the other in cold, the former will experience a sensation of heat and the latter one of cold, although both hands were initially in exactly the same state. It thus appears that the difference in the sensations of heat and cold is due, not only to different states of the organization in which the sensation is produced, but also corresponds to an actual difference in the physical condition of the body by which the sensation is produced. It is with this latter point that we are concerned in the study of heat.

The agency by means of which this difference in the

physical condition is produced is called *heat*, and so long as the *state of aggregation* of a body does not change—that is, so long as it does not change from a solid to a liquid, or from a liquid to a gas—the different conditions due to the agency of heat are referred to as the different *temperatures* of the body. These two terms, temperature and heat, will be more fully understood as we proceed; at present it will be sufficient to notice that temperature of a body, in a given state of aggregation, indicates a particular physical condition of that body, and that heat is the agency to which this condition is due. The relation between heat and temperature is such that when a body gains heat its temperature is said to rise, and when it loses heat, or cools, its temperature falls.

2. Nature of heat. It will now be interesting to consider a little further the nature of this agency which has been distinguished by the name heat. To do so let us examine how heat may be produced. The most familiar source of heat is combustion; this is a chemical process, and its physical nature is too complex to help us at this stage of our investigation. Heat may, however, be produced very simply by mechanical means. When a piece of iron is filed, it is well known that both file and iron become very hot. Also, if a piece of metal be hammered on an anvil, it may be made unpleasantly hot to the touch. Many similar instances will occur to the reader, and in each case it will be found, on consideration, that wherever work is done, or energy disappears, heat is produced. It would thus appear as if there were some relation between the heat developed and the work done during the process of its development. Experiment shows that such a relation does exist, and that it is a constant and definite one. A given quantity of work, if expended in the production of heat, always produces the same quantity of heat, whatever be the process employed for its production, and in all cases it is found that the heat produced is directly proportional to the work expended in its production. From this we may justly infer that **heat is a form of energy**, and that the production of heat by mechanical work is an instance of the

transformation of energy. It may now be asked, What is the nature of this transformation? A bullet is fired against an iron target, and is picked up almost too hot to be held in the hand. What form has the original kinetic energy of the bullet taken in the hot bullet? According to modern views of heat, the molecules of any given body are supposed to be vibrating to and fro, at a rate dependent on the temperature, and the sum total of the kinetic energy of its molecules, due to this vibratory motion, determines the *molecular kinetic energy* for any temperature. As the temperature increases this molecular kinetic energy increases, as does also the potential energy of the system of molecules, and the total change in *molecular energy* accompanying any thermal change is equivalent to the heat generated or absorbed during that change. Hence, in the case of the bullet, its initial kinetic energy is transformed into molecular energy; the motion of the bullet as a whole is stopped by the impact on the target, but the energy of each constituent molecule is thereby increased, and this increase in molecular energy is indicated by the increased temperature of the bullet. Also, the total increase of molecular energy in the bullet and target is equal to the initial kinetic energy of the bullet, and equivalent to the heat developed by the impact.

The process of combustion is capable of a similar explanation. All chemical change is due to molecular or atomic interaction, and it will be understood that the heat effect attending any chemical change may be due to a change in molecular energy resulting from this interaction.

3. Heat is transferable. Many simple experiments show this to be the case. When a kettle of cold water is placed on a fire, heat is absorbed by the water, and ultimately its temperature rises to the boiling point. Or, if a pint of hot water be mixed with a pint of colder water, a transfer of heat takes place, and the temperature of the mixture will be intermediate between the initial temperatures of its components; for it will feel cold to a hand previously placed in the hot water, and hot to one previously placed in the cold water. This shows that, while the hot water has lost heat,

the cold water has gained heat. This simultaneous heating of a cold body and cooling of a hot body during the same operation may be described as the transference of heat from the hot body to the cold one. Knowing these facts, we are led to infer that heat is a physical quantity capable of accurate measurement.

4. Heat a measurable quantity. A quantity is that which can be expressed in terms of a fixed unit of its own kind, and its measure is the ratio of the given quantity to the chosen unit. Thus, if 2 be the measure of any quantity, the meaning is that the quantity considered is twice as great as the fixed unit—*e.g.*, if a foot be chosen as unit of length, a length, the measure of which is 2, is two feet long. Let us now consider if heat is, in this sense, a measurable quantity. Suppose we have a hydrogen gas flame whose rate of burning is absolutely uniform, we may assume that its heating effect is equal for equal intervals of time. If now a fixed quantity of water, at a given temperature, be heated by this flame for a given time, its temperature will be raised to a certain degree, indicating the absorption of a definite quantity of heat. An exactly equal quantity of water will, under the same circumstances, reach the same temperature in the same time, and will therefore absorb the same quantity of heat. Hence, if double the quantity of water be heated under precisely similar circumstances, it will be raised to the same temperature in double the time—that is, after the absorption of double the quantity of heat. If, therefore, the quantity of heat absorbed in either of the cases first considered be taken as the unit, the measure of the quantity absorbed in this last case is 2. Heat may thus be considered as a measurable quantity. The unit adopted in its measurement is quite arbitrary. We might, for example, take the unit to be the quantity of heat required to raise one pound of water from one particular temperature to another. Then the measure of the quantity required to raise m pounds through the *same* range of temperature would be m . This subject will be further considered in the chapter on Calorimetry.

5. Effects of heat. The effects of heat on different substances are very diverse, and often depend on the circumstances under which the heating takes place. We can, however, notice a few of the more important effects. These are:—

(i) **Change of temperature.**

(ii) **Expansion.** It is found that, in general, when a body is heated it expands, and will, if allowed to cool to its original temperature, gradually contract to its original volume. This is the case whether a body be solid, liquid, or gaseous. Some few substances are exceptions to this rule; among them are iodide of silver, stretched caoutchouc (in the direction of the tension), water below 4°C. , and some alloys. These contract when heated and expand on cooling. The amount of expansion is different for different substances; in general, gases expand more than liquids and liquids more than solids for the same rise of temperature. These facts may be illustrated by the following simple experiments:—

1. *Expansion of solids.*

(a) A rod of metal, C D (Fig. 1), passes through holes in



Fig. 1.

two metallic uprights, A and B. The end C is fixed by means of the screw in B, while the end D passes freely through the hole in A. The lever, DOE, is arranged, as indicated in the figure, to magnify the motion of the end D of the rod. The rod is heated by setting fire to some methylated spirit placed in the trough, MN. As the temperature of the rod increases, the upward motion of the lever indicates a gradual increase in length; before long the temperature reaches its maximum, and the lever

remains steady for a short time. As the flame dies away the temperature decreases, the rod contracts, and the lever gradually falls to its initial position as the rod returns to its initial temperature.

(b) Gravesand's Ring (Fig. 2). A sphere and a ring are made of the same metal, and of such dimensions that the former just passes through the latter when both are at the same temperature. If the sphere be heated over a large gas flame and placed on the ring, it will not pass through; but if allowed to remain on the ring, it will gradually contract; and

finally, when both ring and sphere are at the same temperature, it will fall through.

2. *Expansion of liquids.*

To show the expansion of liquids the apparatus shown in the figure is convenient (Fig. 3). The bulb is filled up to a mark, *a*, on the stem with the liquid to be experimented upon. It is then placed in a vessel of hot water; as the glass bulb first touches the hot water, it expands before the liquid does, and the result is that the extremity of the liquid column falls to *b*. Before long, however, the liquid itself begins to expand, and as, in general, liquids expand more than solids, the column gradually rises as the temperature increases. If the apparatus is allowed to cool to its original temperature, the extremity of the column in the stem gradually returns to its initial position at *a*, where it becomes stationary as before.

It must be noticed that the rise and fall of the liquid column depend on the relative expansibility of the material of the



Fig. 2.

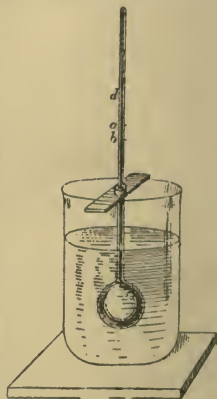


Fig. 3.

bulb and the liquid ; if it were possible to get a liquid of the same expansibility as the material of the bulb, the end of the liquid column would remain stationary through all changes of temperature.

3. *Expansion of gases.*

A bulb (Fig. 4) similar to that used for liquids is taken, and a short thread of mercury introduced into the stem, so as to separate the air in the bulb from the outer air. If now the bulb be held in the hand, or slightly warmed in any other way, the air below at once expands and pushes up the mercury thread, which thus acts as an index serving to show the amount of expansion.

If the expansion of any body be prevented by mechanical force, it will be found that as the temperature increases the force required to prevent expansion will also increase ; and in the case of liquids and solids this force may become enormous. In the case of gases the increase of pressure due to rise of temperature, while the volume of the gas remains constant, follows the same law as the increase of volume with temperature while the pressure remains constant. This fact, which is of great importance, will be referred to again when we come to consider the expansion of gases under the action of heat.



Fig. 4.

It will be seen from the experiments described above that change of length or volume is simultaneous with change of temperature, and this fact suggests a method of indicating the temperature of a body. For example, suppose an iron rod, a metre long, at the temperature of melting ice, to be gradually heated by any means ; its temperature will rise and its length increase continuously, so that a certain definite length of the rod corresponds to each temperature passed through. Thus a length of 1.0001 metre indicates a particular temperature of the rod which may be denoted by t , and whenever the rod has this length it may be assumed that its temperature is that denoted by t . We may thus, by accurately measuring the length of the

rod under different circumstances, get some comparative idea of its variation of temperature.

(iii) **Change of state of aggregation.** By state of aggregation is meant the state of a body with reference to its existence in the solid, liquid, or gaseous form. As we have seen, the general effect of heat on a body is to raise its temperature and, at the same time, slightly increase its volume. Heat has, however, another effect on substances, which, although attended with change of volume, always takes place at a constant temperature. When water has reached its boiling point, it is well known that, although it may continue to boil, its temperature does not rise,—the only effect of a continued application of heat is to convert the water into steam. Similarly, the application of heat to ice converts it into water, the temperature remaining constant throughout the process of conversion. These phenomena of change of state will be more fully considered further on; at present it will be sufficient to learn that at particular stages in the heating of a solid or liquid the effect of the continued application of heat is to change the solid into a liquid or the liquid into a vapour, and that during this change of state the temperature remains absolutely constant.

(iv) **Heat often tends to promote chemical action.** Combustion is an example of this. Carbon and oxygen, if heated to a sufficiently high temperature, in contact with one another unite chemically—that is, combustion takes place. Similarly, sulphur burns in oxygen if the temperature is high enough, and oxygen and hydrogen unite to form water when the mixture is raised to the temperature of ignition by means of an electric spark or lighted taper.

(v) **Heat also modifies in many ways the properties of matter.** The elasticity and rigidity of solids and the viscosity of liquids change, generally in the sense of decrease, as the temperature rises. The conductivity of metals for heat and electricity is also affected by changes of temperature. Also, under proper conditions, the application of heat may give rise to an electric current. These questions

are somewhat beyond the scope of our present work. It is, however, well to remember that *all* the properties of any substance are liable to change with change of temperature. Also, since heat is a form of energy, we may expect that under suitable conditions it may undergo any one of the transformations of which energy is susceptible.

CHAPTER II.

THERMOMETRY.

6. Temperature. It is now necessary to state more accurately what we mean by temperature. We have seen that, when two bodies at different temperatures are placed in thermal communication with each other, there is a transfer of heat from the hot body to the cold one. If, then, we take a body, at some fixed temperature, as a standard (*e.g.*, the temperature of melting ice), and place other bodies in contact with it, it will be found that the transfer of heat will be to the ice if the body is hotter than the ice (a positive transfer), and from the ice if the body is colder than the ice (a negative transfer), and that the amount of this transfer will depend on the difference between the temperature of the body and the standard temperature. We may, then, define the temperature of a body, on any scale, to be *the thermal state of that body with reference to its power of communicating heat to a substance kept at a standard temperature which is the zero of the scale.* In this way all temperatures higher than zero must be considered positive and all below negative, because in one case the transfer of heat to a substance kept at the standard temperature would be positive and in the other negative. As a direct consequence of this definition of temperature, we may state that *difference of temperature is the condition of flow of heat from one body to another*, just as difference of level is the condition of flow of water from one point to another. The direction of the flow is always in the direction of the fall of temperature. Substances are said to be at the same temperature when no transfer of heat takes place, when they are put in contact with one another.

7. Preliminary idea of a thermometer. We have seen that it is possible to get some idea of the temperature of a body by taking note of its change of volume, so long as this change of volume is simultaneous with change of temperature. When we further remember that heat may be transferred from one body to another, the transfer taking place from the hot to the cold body until they reach a *common temperature*, we see that one body may not only indicate its own temperature, but also that of any other body with which it is in contact. For example, the iron rod mentioned in Art. 5 might be taken into the different rooms of a house, and, if sufficient time be allowed in each case for it to take the temperature of the room, its length in any particular room would be a definite indication of the temperature of that room. If an apparatus similar to that shown in Fig. 3 be placed in a vessel of water at the ordinary temperature, the extremity of the liquid column will, in a short time, take up a fixed position in the stem of the bulb. This position, which may be denoted by A, corresponds to the common temperature of the bulb and the water in which it is placed. Let the apparatus be now placed in a vessel of hot water; the top of the column will, after a slight fall due to the sudden expansion of the bulb, gradually rise and finally take up another fixed position, B, in the stem. This position again corresponds to the common temperature of the bulb and the water in which it is placed. Now to each position intermediate between A and B there corresponds a definite temperature, and this suggests the idea that by carefully graduating the stem we may establish an arbitrary scale of temperature, to which the temperature of any body, within the limits of our scale, may be referred. Any apparatus constructed on these principles may be called a thermometer.

8. Fixed points of reference on a thermometric scale. On consideration it will be seen that the scale of temperature referred to in the preceding article is altogether arbitrary. The liquid in the bulb is not specified, and the upper and lower points are quite indefinite. We have

also seen (Art. 5) that the indications of the expansion of a liquid in a vessel depend on the expansibility of the material of the vessel. Hence, the material of the bulb should also be specified. If, then, a number of different people constructed thermometers in the way described above, the temperature indications given by each instrument would not admit of comparison except by direct experiment. This difficulty will, however, be solved to some extent if it be decided to employ a particular liquid in a bulb of specified material, and if two standard temperatures, which can be easily produced and referred to, are chosen as fixed points on the scale. For reasons which we shall consider, when prepared to do so, the liquid most generally employed in the construction of thermometers is mercury, and the two temperatures taken as points of reference are the temperature of melting ice and the temperature of the steam given off by water boiling under the normal atmospheric pressure. It is found that the temperature at which ice melts at the normal atmospheric pressure is perfectly constant; it varies very slightly with the pressure, but the variation for ordinary cases is quite negligible. The temperature of the steam given off by boiling water depends, as we shall see, on the pressure at which boiling takes place; hence, it is necessary to define the pressure in choosing this standard temperature. If, now, the apparatus of Fig. 3, having mercury in the bulb, be placed in a vessel and quite surrounded with melting ice, the end of the column in the stem will become stationary at a certain point, A. If it next be taken out and placed in the steam from water boiling under normal atmospheric pressure, the end of the column will become stationary at another point, B, in the stem. These two points, A and B, correspond to definite temperatures, which can be reproduced and referred to at any time or in any place. It only remains now to decide upon a plan of graduation of the stem between and beyond the points A and B, and to devise means for the accurate carrying out of this plan, in order to obtain a practical scale of temperature comparable in its indications with other scales similarly constructed.

9. The mercurial thermometer (Fig. 5). We shall now consider, more in detail, the construction of a mercurial thermometer, the general idea of which has been developed in the preceding paragraphs. This instrument consists essentially of a glass tube called the *stem*, of very fine bore and thick walls, and terminating in a reservoir with thin walls which, whatever its shape may be, is called the *bulb*. The bulb and part of the stem are filled with mercury, and the variations in temperature are indicated by the change of position of the end of the column of mercury in the stem, referred to a scale attached to, or engraved on, the stem. For accurate purposes the scale is always engraved on the stem.

Construction of the thermometer. A length of thick capillary tubing, having as uniform a bore as can be found, is taken and well washed, first with nitric acid, then with water, and finally dried by a current of hot air. Before proceeding to construct the instrument, it is well to make a preliminary test of the uniformity of the bore of the tube chosen. For this purpose a short thread of mercury about an inch long is introduced into the tube. This is done by attaching to one end of the tube a small india-rubber ball (Fig. 6).



Fig. 6.



Fig. 5.

By squeezing the air out of the ball and dipping the other end of the tube into mercury, a small quantity will rise into the tube, and, with a little management, a thread the desired length may be got into the bore and moved into any part of the tube. This done, the length of the thread is measured by suitable means in a particular position in the tube; the thread is then moved into some other position and again measured, and so on. If the length remains constant for different positions, it is evident that the bore must be perfectly uniform. This is never found to be the case with any tube and if the differences in length are very great the tube must be

rejected and another tried. When finally a suitable tube is obtained, one end is heated till melted, and is then blown into a bulb by forcing air in at the other end of the tube by means of an india-rubber ball. The shape of this reservoir bulb is different for different instruments, but the most usual and convenient form is a cylinder of the same external diameter as the tube. The size will depend on the purpose the thermometer is intended to serve. The larger the reservoir the longer will be the degrees of the scale. In order to introduce the mercury into the reservoir a bulb is blown at the upper end of the tube and drawn out into



a fine point (Fig. 7). This bulb, and also the reservoir, are gently heated and then allowed to cool with the drawn-out point immersed in a vessel of mercury ; as the air inside the bulbs gradually cools, the pressure becomes less than the external atmospheric pressure, and a quantity of mercury is forced up into the nearer bulb. The tube is now inverted, and by alternate heating and cooling of the reservoir a portion of the mercury in the upper bulb descends into the lower. This being done, the tube is next laid on a sloping furnace, arranged so as to heat it uniformly throughout its entire length. In this position the mercury boils, and its vapour expels the air, which bubbles through the mercury in the upper bulb. All traces of air and moisture are thus removed, and when the tube is allowed to cool the mercury in it forms a continuous column, filling the reservoir, stem, and portion of the upper bulb. The excess of mercury in the last is poured off, and the tube is then heated a little above the highest temperature it is intended to indicate. The mercury now completely fills the stem, and while it does so a blow-pipe flame is brought to bear upon the tube just below the temporary bulb, which is thus fused off and the thermometer finally closed. The instrument is now ready for graduation ; but it is better to keep it from four to six months before doing so, because when glass has been heated to a high temperature it takes a long time (in some cases, years) to recover its original volume.

As a preliminary to graduation it will be necessary to mark on the stem the position of the thread corresponding to the two standard temperatures referred to above. This process is known as the determination of the fixed points.

Determination of the fixed points. (a) Freezing point. To determine this point the thermometer is placed in a cylindrical vessel (Fig. 8), and the reservoir and the lower part of the stem are completely surrounded by melting ice. When the level of the mercury becomes perfectly stationary,



Fig. 8.

a fine mark is made on the stem at the extremity of the column. This is one of the fixed reference points on the scale, and indicates the temperature of melting ice.

(b) Boiling point. In order to mark this second point on the thermometer the apparatus shown in Fig. 9 is used. It is made of copper, and the steam from water boiling in the lower part circulates round the double casing of the upper part, and finally escapes by a side tube near the bottom of the outer casing. The thermometer is passed through a cork, and placed in the apparatus in the position indicated in Fig. 9. After some time the mercury becomes stationary, and the

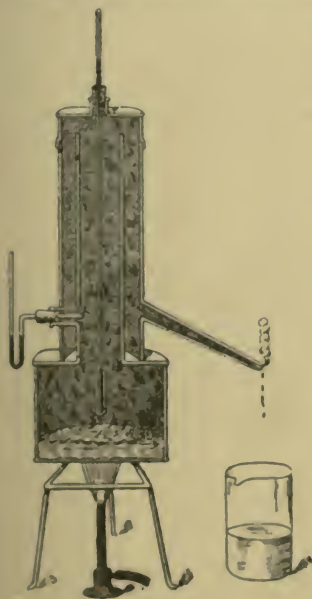


Fig. 9.

thermometer is adjusted so that the top of the mercury column is just seen above the cork; its position is then marked, and the second reference point on the scale is

thus determined. It indicates the temperature of steam from water boiling under the atmospheric pressure indicated by the barometer at the time of the determination. For the present, we shall suppose this to be normal.* It should be noticed that the thermometer is placed in the steam and not in the boiling water. It has been found that the temperature of boiling water depends on a variety of circumstances; whereas the temperature of the steam depends only on the pressure at which boiling takes place.

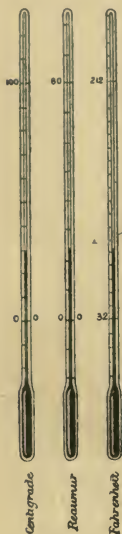


Fig. 10.

Graduation. The two reference marks being thus obtained, it remains to graduate the space between them, and to continue the division throughout the whole length of the stem. A zero point is then chosen, and the divisions below it are marked negative, and those above positive (Art. 6). There are three different scales of graduation now in use; and as we may have temperatures expressed in any one of these scales, it will be necessary to consider them. They are called the Centigrade, Fahrenheit, and Réaumur scales (Fig. 10).

For the Centigrade scale we are indebted to Celsius. On it the freezing point is taken as zero, and marked 0° ; the boiling point is marked 100° . The space between the fixed points is thus divided into 100 equal divisions, called degrees. Graduation on the same scale is extended beyond the reference points; and divisions below zero—*i.e.*, below freezing point—are considered negative, and those above, positive. This scale is the one most generally used for scientific purposes.

The Fahrenheit scale was introduced by Fahrenheit of Dantzic, about the year 1714. On it the freezing point is marked 32° , and the boiling point 212° , the space between being divided into 180 equal degrees, and the division

* The normal atmospheric pressure is that indicated by a height of 760 mm. in the mercury barometer.

extended above and below the fixed points. In this method of division the zero is at a point 32° below the freezing point; it is supposed to indicate the lowest temperature which can be got by mixing snow and salt. This scale is very generally used for the purposes of ordinary life, and, to some extent, for those of science.

On the Réaumur scale the freezing point is taken as zero, and marked 0° , and the boiling point is marked 80° . There are thus 80 equal divisions between the fixed points on this scale. It is used in several European countries for medical and domestic purposes.

We have spoken above of dividing the space between the fixed points into so many *equal* divisions. This must be understood to mean that the *volume* of the bore between any two consecutive divisions is equal. If the bore were perfectly uniform, it would only be necessary to have the divisions of equal length; and in ordinary cases this is how the division is effected. For accurate purposes, however, it will be necessary to *calibrate* the tube. This is a complicated process, into the details of which we cannot here enter. The principle of the method employed is the same as that which we have seen applied in testing the uniformity of the bore of the thermometer tube. A short thread of mercury, *ab*, the volume of which is necessarily constant, is moved from one end of the tube to the other in such a way that the end *a* is, at each move, brought into the exact position previously occupied by *b*. Its length is accurately measured in each successive position, and then, by the combined help of calculation and the mechanical adjustments of a dividing engine, the stem may be divided so as to indicate uniform increase of volume of the contained mercury. It will be noticed that this method of graduation assumes that mercury expands uniformly throughout the whole range of temperature indicated by the thermometer; if the expansion is not uniform, a degree on one part of the stem may not mean the same as a degree on any other part—*e.g.*, expansion from 98° to 99° on the Centigrade scale may not indicate the same rise of temperature as that from 1° to 2° on the same scale. Experiment has shown that the expansion of mercury between

the fixed points is very nearly uniform, and this is one of the reasons why it has been chosen as the standard thermometric substance.

11. Different forms of thermometers. For special purposes various forms of thermometers are used. The more important of these are:—

1. Alcohol thermometer. Alcohol is sometimes used instead of mercury in the construction of thermometers. It has the advantage that it can be employed for very low temperatures: mercury solidifies at -39°C ., whereas alcohol may be exposed to a very much lower temperature without solidifying. It also expands much more than mercury for the same rise of temperature, but the expansion is not uniform; it increases with the temperature, and for this reason alcohol thermometers are usually graduated by comparison with a mercurial thermometer.

2. Maximum and minimum thermometers. It is often necessary to know the highest or lowest temperature reached in a given interval of time. For instance, it is usual in making meteorological observations to record the highest (maximum) temperature attained during the day, and the lowest (minimum) temperature reached during the night. For these purposes maximum and minimum thermometers are employed.

Six's maximum and minimum thermometer is one of the oldest of its class, and is at once a maximum and a minimum thermometer. Its general plan of construction is shown in Fig. 11. The bulb B, and part of the stem down to *m* is filled with alcohol. This is the real thermometric part of the instrument: the column of mercury, *mm'*, which occupies the U-shaped part of the tube merely acts as an index. The cup C (which is open to the air), together with a part of the



Fig. 11.

stem down to m' , contains alcohol, the use of which is merely to protect the index i' . As the alcohol in B expands or contracts the extremities, m and m' , of the mercury column rise or fall as the case may be, and the extreme point reached in each case is indicated by one end of a light steel index, i or i' (shown on a larger scale at I). This index is pushed before the mercury column, and is prevented from returning by means of a spring, s , which is just strong enough to hold it in its place. The position of the index i evidently indicates the minimum temperature reached, and that of i' the maximum temperature. When it is required to set the instrument ready for any observation, these indices are brought into contact with the ends m and m' of the mercurial column by means of a small magnet. This instrument is largely used by gardeners and nurserymen.

Rutherford's maximum and minimum thermometers are two separate instruments, but are usually mounted on the same frame, as shown in Fig. 12. The maximum thermo-



Fig. 12.

meter, X, is a modification of the ordinary mercurial thermometer placed in a horizontal position. The maximum temperature is registered, as in Six's instrument, by means of a steel index, and the method of setting previous to taking an observation is the same. As the thermometer lies horizontally, there is no necessity to attach a spring to the index. The maximum temperature will thus be indicated by the position of that end of the index which is nearest the bulb of the thermometer.

The minimum thermometer, N, is an alcohol thermometer, and is the only minimum thermometer in general use. It is placed in a horizontal position, and for registering the minimum temperature there is a small index of glass or enamel, which allows the column of alcohol to

expand past it without moving it; but when the alcohol contracts, the index, being wetted by the liquid, is drawn backwards by the surface film at the extremity of the alcohol column. The minimum temperature is thus indicated by the position of that end of the index which is furthest from the bulb. The instrument is set by inclining it in such a way as to cause the index to slide down to the end of the liquid column.

There is one important form of thermometer which we have not mentioned in this article. It is the *air thermometer*. When we come to consider what should be the properties of a perfect thermometric substance, we shall find that air (or some similar gas, which closely conforms to Boyle's Law) comes nearest the required standard, and is sometimes used when very accurate measurements are required. More usually a mercurial thermometer, which has been compared with an air thermometer, is employed. We shall consider the air thermometer when dealing with the expansion of gases.

12. Sources of error in a mercurial thermometer. The more important sources of error are:—

1. *Change of zero.* In many cases, if an ordinary mercurial thermometer, which has been made for some time, be placed in melting ice, it will be found that its indication is a little higher than the freezing point marked on the scale. The reason of this is referred to in Art. 9. If the tube has not been kept long enough before graduation, the gradual contraction of the bulb after graduation causes its interior volume to decrease, and consequently the volume of the contained mercury at 0° , although constant, appears to increase, and the level of the column corresponding to the freezing point rises. To correct this error, it is only necessary to redetermine the freezing point from time to time, and to deduct the observed error from any reading made. Thus, if zero is found, some months after graduation, to be at 0.1°C. , and the reading of the thermometer when placed in a bath of water is 64.5°C. , the correct temperature of the bath is 64.4°C.

2. *Recent heating.* This is a temporary source of error exactly similar to that described above. If a thermometer be placed in steam, or otherwise heated to a high temperature, and then allowed to recover its original temperature, the bulb will not for some weeks recover its original volume; hence, if placed in ice, the freezing point on the scale will appear to be too high, owing to the temporary increase in the volume of the bulb. It is for this reason that, in the construction of thermometers, the freezing point is determined before the boiling point; if the latter were determined first and the former soon after, the zero would be marked too low on the scale, and would gradually rise, as explained above under *change of zero*. No correction is usually applied in this case, as the error is merely temporary, and is, after a day or so, quite inappreciable. Care must, however, be taken not to use the same thermometer for determining low temperatures soon after it has been used for high temperatures.

3. *Temperature of steam at time of marking boiling point.* The temperature of steam depends on the atmospheric pressure under which boiling takes place. At 760 mm. it corresponds to 100° C. or 212° F.; but if the pressure be greater or less than 760 mm., the boiling point indicates a temperature higher or lower than this temperature.

Tables have been drawn up from experiments by Regnault giving the boiling points corresponding to a wide range of pressures. Hence, if the barometric height be read at the time of determining the boiling point, the temperature corresponding to this height can be obtained from these tables, and the graduation carried out accordingly.

4. *Temperature of the stem.* When a thermometer is used to determine a temperature, it is often impossible to subject more than the bulb and a small portion of the stem to this temperature. The upper portion of the stem is thus at a different temperature from the rest of the tube, and consequently the reading will be slightly different from what it would be if the whole thermometer took up the temperature it is intended to indicate. If a thermometer indicate

a temperature T when placed in a bath, and have n divisions of the mercury column *outside* the bath, at a temperature t , then the true temperature of the bath, T' , is given by—

$$T' = T + n(T' - t)c_a^*,$$

where c_a is the mean co-efficient of apparent expansion of mercury in glass. (For proof of this correction see Ex. V. 4. It is a simple problem in expansion.)

5. A thermometer should always be read in the position in which it was graduated ; for if the bore be very fine and the bulb thin, the influence of the weight of the contained mercury on the volume of the bulb may cause an appreciable change of the reading when the position of the instrument is altered.

6. Theoretically, corrections should be made for the pressure of the air at the time of determining freezing point, and also for the influence of external pressure on the volume of the bulb ; but these sources of error are, under ordinary conditions, quite inappreciable.

* The difference between T and T' is so small that the more convenient formula, $T' = T + n(T - t)c_a$, may be used with sufficient accuracy.

CALCULATIONS.

13. It is often necessary to convert temperatures expressed in one scale into the corresponding temperatures on either of the other scales. In doing so there are two things to be noticed: (1) Since the interval of temperature between the freezing and boiling points is constant, it follows that 180 Fahrenheit degrees = 100 Centigrade degrees = 80 Réaumur degrees. (2) The zero of the Fahrenheit scale is 32 degrees below freezing point—*i.e.*, 40° F. indicates a temperature 8 degrees F. above freezing point. If, therefore, F, C, and R denote corresponding readings on the Fahrenheit, Centigrade, and Réaumur scales respectively, we have that (F - 32), C and R denote, in each case, the number of degrees the given temperature is above freezing point. Hence, from (1) above—

$$(F - 32) : C : R :: 180 : 100 : 80.$$

That is—

$$(F - 32) : C : R :: 9 : 5 : 4.$$

This proportion may be written thus—

$$\frac{(F - 32)}{9} = \frac{C}{5} = \frac{R}{4}.$$

This should be remembered, and applied, in all cases where it is required to convert temperatures from one scale into another.

EXAMPLES I.

1. Find the temperature, on the Fahrenheit scale, corresponding to 40° C.

Here the two scales involved are Fahrenheit and Centigrade. Hence, we write—

$$\frac{F - 32}{9} = \frac{C}{5}.$$

Substituting 40 for C, we have—

$$\frac{F - 32}{9} = \frac{40}{5} = 8.$$

Or—

$$F - 32 = 72.$$

Therefore—

$$F = 104.$$

That is, 40° C. corresponds to 104° F

2. Find the temperature, on Réaumur's scale, corresponding to -40° F.

As above—

$$\frac{F - 32}{9} = \frac{R}{4}.$$

Here—

$$\frac{-40 - 32}{9} = \frac{R}{4}.$$

Or—

$$\frac{-72}{9} = \frac{R}{4}.$$

That is—

$$\frac{R}{4} = -8.$$

Or—

$$R = -32.$$

3. Find what temperature, on Fahrenheit's scale, is represented by the same number on the Centigrade scale.

Again—

$$\frac{F - 32}{9} = \frac{C}{5}.$$

Let T denote the required temperature.

Then—

$$\frac{T - 32}{9} = \frac{T}{5}.$$

Or—

$$5T - 160 = 9T$$

$$4T = -160. \quad \therefore T = -40.$$

Hence -40° C. corresponds to -40° F.

4. Find the temperatures on each of the two other scales corresponding to—

- (1) -70° C. (2) 76° F. (3) -24° R. (4) 0° C. (5) 50° F.
 (6) 68° F. (7) 64° R. (8) 92° C. (9) 14° F. (10) 50° R.

14. Before reading the following chapters on expansion, it will be well for the student to thoroughly master the following points:—

1. The square, or cube (or higher power) of any *small* quantity, or the product of two small quantities, is negligibly small.

Thus—

$$(\cdot 0002)^2 = \cdot 00000004.$$

$$(\cdot 0002)^3 = \cdot 000000000008.$$

Also—

$$\cdot 0002 \times \cdot 0003 = \cdot 00000006.$$

It is evident that if the quantities $\cdot 0002$ and $\cdot 0003$ are *small* compared with any quantity, then these three products are *negligibly small* compared with the *same* quantity.

2. Suppose a and β to be small quantities, *compared with unity*, and consider the following relations—

$$(1 + a)^2 = 1 + 2a + a^2.$$

Now, by (1), a^2 is negligible; hence—

$$(1 + a)^2 = 1 + 2a;$$

and, similarly—

$$(1 + a)^3 = 1 + 3a + 3a^2 + a^3 = 1 + 3a.$$

Also—

$$(1 + a)(1 + \beta) = 1 + a + \beta + a\beta = 1 + a + \beta$$

($a\beta$ being negligible as the product of two small quantities)

$$\frac{1}{1 + a} = 1 - a + a^2 - a^3 + \text{etc.}, \text{ by actual division, i.e., } \frac{1}{1 + a} = 1 - a.$$

$$\frac{1 + a}{1 + \beta} = 1 + a - \beta - a\beta + \text{etc.}, = 1 + a - \beta.$$

It may be useful to tabulate these *approximate* results for future use and reference.

$$(1 + a)(1 + \beta) = 1 + a + \beta.$$

Or generally—

$$(1 \pm a)(1 \pm \beta) = 1 \pm a \pm \beta.$$

$$(1 \pm a)^2 = 1 \pm 2a.$$

$$(1 \pm a)^3 = 1 \pm 3a.$$

$$\frac{1}{1 + a} = 1 - a; \quad \frac{1}{1 - a} = 1 + a.$$

$$\frac{1 + a}{1 + \beta} = 1 + a - \beta.$$

EXAMPLES II.

Find the approximate value of—

$$(1) \quad (1 \cdot 000024)(1 \cdot 000065). \quad [(1 + a)(1 + \beta)].$$

$$(2) \quad (1 \cdot 00018)(\cdot 99982). \quad [(1 + a)(1 - \beta)].$$

$$(3) \quad (1 \cdot 00035)^2; (\cdot 999987)^3. \quad [(1 + a)^2, (1 - a)^3].$$

$$(4) \quad \frac{1}{1 \cdot 000025}; \quad \frac{1}{0 \cdot 99987}. \quad \left[\left(\frac{1}{1 + a} \right), \left(\frac{1}{1 - a} \right) \right].$$

$$(5) \quad \frac{1 \cdot 00016}{1 \cdot 00004}; \quad \frac{1 \cdot 00018}{0 \cdot 99986}. \quad \left[\frac{1 + a}{1 + \beta}; \quad \frac{1 + a}{1 - \beta} \right].$$

$$(6) \quad 100 \left[\frac{1 + 10(\cdot 000064)}{1 - 25(\cdot 000008)} \right]; \quad \frac{100}{1 \cdot 00016}. \quad \left[\frac{n}{1 + a} = n \frac{1}{1 + a} \right].$$

CHAPTER III.

EXPANSION OF SOLIDS.

15. WE have seen that, in general, all bodies expand under the influence of heat. It is now necessary to deal with this question more in detail, and to consider the experimental methods which have been adopted in investigating it. We shall begin by considering the expansion of solids. When a solid body expands we have, simultaneously with the increase of volume, a corresponding increase in each dimension. If we consider the expansion of one dimension (*e.g.*, length), we have *linear expansion*; but when we consider the expansion of a body as a whole—that is, its increase of volume—we have *cubical expansion*.

16. **Coefficient of expansion.** Suppose a body, of volume V_0 at 0°C. , to be heated to $t^\circ \text{C.}$, and that its volume at this temperature is denoted by V_t . Then, the increase of volume for $t^\circ \text{C.}$ rise of temperature $= V_t - V_0$. Hence, the average increase for 1°C. is $\frac{V_t - V_0}{t}$, and the increase of volume, per *unit* volume at 0° , for 1°C. rise of temperature is $\frac{V_t - V_0}{V_0 t}$. This is the *mean coefficient of cubical expansion of the body between 0° and t°* . It may be defined, for any substance, between given limits of temperature, as the ratio of the average increase of volume of a mass of that substance, for 1°C.^* rise of temperature, to the volume of the

* Coefficients of expansion are generally defined with reference to the Centigrade scale, but any scale may be chosen. If c denote the coefficient of expansion of a given substance, referred to the Centigrade scale, then $\frac{5}{9}c$ and $\frac{1}{273}c$ represent the coefficient of expansion for the same substance referred to the Fahrenheit and Réaumur scales respectively.

same mass at 0°C . Let it be denoted by c . Then we have $\frac{V_t - V_o}{V_o t} = c$, or, $V_t - V_o = V_o ct$. $\therefore V_t = V_o(1 + ct)$. This formula establishes a relation between V_o , V_o , c and t , which serves to determine any one when the others are given.

Similarly, if L_o and L_t denote the lengths of a rod at 0°C . and $t^\circ \text{C}$., we have, in exactly the same way, $\frac{L_t - L_o}{L_o t} = l$, where l is the *mean coefficient of linear expansion* between 0°C . and $t^\circ \text{C}$. This coefficient may be defined, for any substance, between given limits of temperature, as the ratio of the average increase of length of a bar of that substance, for 1°C . rise of temperature, to the length of the same bar at 0°C . As in the case of cubical expansion we have between L_o , L_o , l and t , the relation $L_t = L_o(1 + lt)$.

17. Relation between linear and cubical expansion. Suppose we have a cube of metal, of unit edge, and of unit volume. Let its temperature be raised through 1°C . Then, using the notation of preceding article, the length of each edge would be $(1 + l)$, and the volume becomes $(1 + c)$. But the volume of a cube of edge equal to $(1 + l) = (1 + l)^3$. Hence, we may write $(1 + c) = (1 + l)^3$. Now, experiment has shown that l is a very small quantity, compared with unity. Hence [Art. 14 (2)] we have $1 + c = (1 + l)^3 = 1 + 3l$, or $c = 3l$. That is, the mean coefficient of cubical expansion of a solid is approximately equal to three times the mean coefficient of linear expansion of the same solid.

18. Experimental determination of linear expansion. Many methods have been devised for determining the linear expansion of different substances. All the methods have some points in common which it will be well to notice.

1. In each method a bar of the substance, whose expansion is to be determined, is placed in a trough, and first surrounded with melting ice to give it a temperature of 0°C ., and then raised to a temperature $t^\circ \text{C}$. by covering it, in the trough, with water at $t^\circ \text{C}$.

2. The length of the bar is accurately measured at 0°C. and $t^{\circ}\text{C.}$ The difference of the measurements thus taken gives the expansion of the bar for a rise in temperature of $t^{\circ}\text{C.}$

This difference is very small, and hence very accurate measurement is required, and it is in the means adopted to secure accuracy that the apparatus used by different experimenters chiefly differ.

Two methods of measurement are possible. (a) To employ very accurate instruments specially adapted to the measurement of small lengths, and thus measure the increase in length directly. (b) To mechanically magnify the increase in length in some known ratio. The magnified indication may then be measured by ordinary means, and the real increase of length deduced from this measurement and the known scale of magnification.

Laplace and Lavoisier employed this latter method, and Roy and Ramsden, in a more reliable research, adopted the direct method.

The method of Laplace and Lavoisier. The principle of the method employed by these experimenters is shown in Fig. 14. The bar, A B, to be experimented on is sup-

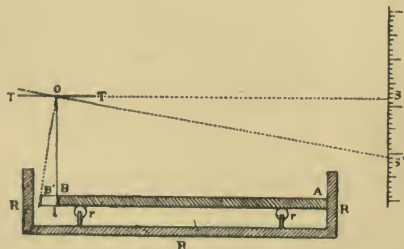


Fig. 14.

ported on rollers, rr , in a trough, RRR . One end, A, abuts on a solid support at one end of the trough: the other end, B, is in contact with a lever, Ol , capable of

rotation about an axis at O and rigidly attached to a telescope, TOT, which can be focussed on the scale, SS'. The end A of the bar is thus fixed, and hence, on expansion, the end B advances, pushing before it the lever, OL, and its attached telescope. In this way a very small motion of the end B causes the line of sight of the telescope to travel over a considerable length of the scale, SS'. To make the determination, the trough, R R R, is filled with water, which is reduced to 0° C. by adding pieces of ice in such a manner as not to interfere with the motion of the bar on the rollers. When the temperature remains stationary at 0° C. (as indicated by thermometers placed in the trough), the telescope is focussed on the scale, and the division appearing in the centre of field of view is noted. Warm water is then put in the trough, and when a uniform temperature, t° C., is attained, the telescope is again focussed, and the reading on the scale noted as before.

To show how the expansion may be calculated from these observations, we may take the simple case shown in the figure. Let the continuous lines denote the positions of the different parts of the apparatus at 0° C., and let the dotted lines denote the changes in position due to expansion. Then from the readings on the scale the distance, SS', is known, and it is required to find BB'. The triangles BOB' and SOS' are similar; therefore we have $\frac{BB'}{SS'} = \frac{OB}{OS}$. The distances OB and OS can be measured, and, as SS' is known, BB' can be calculated. The coefficient of linear expansion is given by $\frac{BB'}{AB \times t}$; BB' is determined as above, AB must be measured at 0° C., and t is given by the thermometers in the trough. Laplace and Lavoisier measured the coefficient of linear expansion of a great number of solids by this method.

An objection to the method is that it is very difficult to determine the ratio $\frac{OB}{OS}$ with any accuracy.

The measurement of the cubical expansion of solids is usually an indirect process, and will be considered after the expansion of liquids has been dealt with.

19. **Practical precautions necessary on account of expansion of metals by heat.** The expansion of metals is very frequently a source of trouble. Thus, in constructing a railway, small spaces have to be left between the metals to allow for expansion due to the extreme yearly range of temperature. Similarly, allowance must be made for expansion in constructing an iron bridge, and it is often difficult to secure rigidity and yet allow each part freedom of expansion. The tubular girders of the Britannia Bridge are mounted on rollers at each end, so that they may be free to expand, without straining the structure. For the same reason, the joints of water pipes and gas mains are made like those of a telescope. If furnace bars are firmly fixed, they become bent, owing to the expansion which they undergo on heating; for this reason, the ends of the bars should be free to move in the direction of their length. Metal castings often break in cooling, on account of one part cooling more rapidly than another, and thus producing great strains in the metal. For example, in casting a wheel, with a thin rim but massive arms, the rim solidifies and cools much more quickly than the arms, and as a consequence the contraction of the latter is opposed by the rim, and a stress is set up which may fracture the rim or one or more of the arms. To avoid this, the arms are often bent, so that the strain thus produced is relieved as they straighten. In all measurements, made by metal rods or chains, the temperature must be noted, and a correction applied. The imperial standard yard is the distance, at 62° F., between two fine lines engraved on gold plugs, which are inserted into a bronze bar kept in the Exchequer Chamber.

In the construction of clocks, chronometers, and watches of the best kind, great precaution has to be taken to obviate the effects of expansion on the time-keeping part of the mechanism. The rate of a clock is generally controlled by a pendulum, whose time of vibration varies as the square root of its length, or, more accurately, as the square root of the distance between its centres of oscillation and suspension. If the temperature varies this distance varies, and the clock goes 'slow' as the temperature rises

and "fast" as the temperature falls. Compensating pendulums are constructed to prevent this change of rate with change of temperature.

Harrison's gridiron pendulum (Fig. 18) is the best-known form. It consists of four oblong frames arranged as shown in figure. The vertical rods are alternately of steel (S) and brass (B). The central steel rod passes through holes in the lower horizontal pieces of the frames, and carries the bob at its lower end. The arrangement is such that the steel rods expand downwards, while the brass rods expand upwards; and if the downward and upward expansions be equal, the centre of the bob is neither raised nor lowered. If it were possible to have the total length of each set of rods the same, only one metal would be required, but the inner rods must necessarily be shorter than the outer ones, and therefore, if compensation is to be exact, they must be made of the more expansive metal. The condition of compensation therefore requires that the total expansion of one set of rods should be equal to the total expansion of the other, the rods being so arranged that these expansions take place in opposite directions. Let L_o denote the total length at 0°C . of one set of rods, and L'_o the total length of the other set; then—

$$L_t = L_o (1 + \alpha t), \text{ and } L'_t = L'_o (1 + \beta t),$$

where α and β are the corresponding coefficients of linear expansion. Hence, in one case, the expansion is given by—

$$L_t - L_o = L_o \alpha t,$$

and in the other by—

$$L'_t - L'_o = L'_o \beta t.$$

For compensation we must have—

$$L_o \alpha t = L'_o \beta t, \text{ or } L_o \alpha = L'_o \beta, \text{ or } \frac{L_o}{L'_o} = \frac{\beta}{\alpha}.$$

That is, the total effective lengths* of the rods in each set

* It must be here noticed that the two vertical bars of each frame only effect the same expansion of the frame as one bar of the same

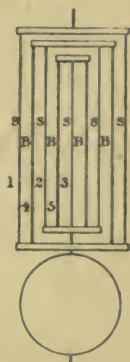


Fig. 18.

must be taken inversely proportional to the mean coefficient of expansion of the metals of which they are made. For

steel and brass the ratio $\frac{\beta}{\alpha} = \frac{2}{3}$ nearly.



Fig. 19.

Graham's mercurial pendulum (Fig. 19). The bob of this pendulum consists of a frame carrying two glass cylinders containing mercury. The principle of compensation is the same as that described above; the rod to which the bob is attached expands downwards, while the mercury expands upwards, and, if the quantity is properly adjusted, the final result is that the distance between the centres of oscillation and suspension is unchanged, and therefore the rate of the clock is unaffected by changes of temperature.

In chronometers and watches the rate is controlled by balance wheels, which oscillate under the influence of a steel hair-spring. The time of oscillation increases if the dimensions are increased, and therefore a rise of temperature causes the rate of the watch to diminish. Further, a rise of temperature lessens the elasticity of the hair-spring [Art. 5 (v.)], and this also tends to increase the time of oscillation, and diminish the rate of the watch. In chronometers and the best-made watches, compensation is effected by making the circumference of the wheel of two metals, the outer being the more expandible. This double rim is made in three parts, each supported by an arm of the wheel (Fig. 20). The effect of rise of temperature on this arrangement is, that while the



Fig. 20.

length. Hence, in the figure, the effective length of the bars is $(1 + 2 + 3)$ for steel, and $(4 + 5)$ for brass.

end of segment of the rim nearest the supporting arm is pushed out from the centre by the expansion of that arm, the other end curls in towards the centre of the ring owing to the greater expansion of the outer strip of metal. By properly adjusting the screw weights, carried by each segment, exact compensation can thus be obtained.

20. Metallic thermometers. Thermometers have been constructed with metals. They are of little use as accurate instruments, but are convenient for giving an automatic record of the variation of temperature, and thus act as maximum and minimum thermometers.

The best-known instrument of this class is *Breguet's thermometer* (Fig. 21). If two strips of metal of different

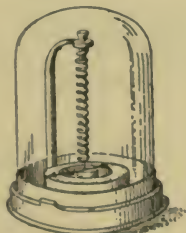


Fig. 21.

expansibility be riveted or soldered together and wound into a spiral with the most expansible metal inside, then any rise of temperature causes the inner strip to expand more than the outer, and consequently the spiral unwinds. Similarly, if the temperature falls, the spiral coils up. This thermometer is an application of this fact. It consists of a spiral composed of three metallic strips of silver, gold, and platinum, soldered together so as to form a single ribbon. The silver, which is the most expansible, is placed inside, the platinum outside, and the gold between serves to connect them. The spiral carries, at its lower end, a horizontal needle, which traverses a dial. As the spiral winds or unwinds with change of temperature, the needle is deflected

in a corresponding direction, and indicates, by its motion, the variation of temperature.

21. Force of expansion and its applications. The force exerted by a metal rod, if prevented from expanding or contracting as its temperature changes, is very great. For example, if an iron bar, of one square inch cross-section, be heated from 0°C. to 100°C. , and then fixed, so as to prevent contraction, it will exert a force equal to the weight of about thirteen tons on the fixings. This may be illustrated by the simple apparatus shown in Fig. 22. The bar, A B, is heated

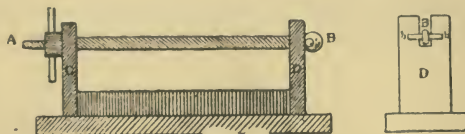


Fig. 22.

by a lamp until it expands sufficiently to allow a short cast-iron rod (*b*) to be passed through the hole at the end B, outside the upright D. The bar is then allowed to cool, and, up to a certain point, contraction is resisted by the cast-iron rod; finally, however, the stress becomes too great, and the rod is broken in two. This contractile force of metals, due to fall of temperature, has many useful applications. The tyres of ordinary carriage and cart wheels are fitted on when red hot, and, on cooling, bind the wheels firmly together. Boiler plates are riveted with red-hot rivets, which, on cooling, draw the plates so close together that a steam-proof joint is formed. In cases where the walls of buildings have bulged outwards, they have been drawn in by passing iron bars through them, across the building, and attaching to their extremities, outside the building, iron plates screwed up to the wall with a nut. The bars are then heated, the nut and plate screwed up tightly against the wall, and the bar allowed to contract, drawing the walls with it. This process is repeated, until the walls are made to assume their proper positions.

22. The mean coefficients of linear expansion between 0° and 100° C. of some substances are given below.

Table of coefficients of linear expansion.

Glass	0.000008
Iron	0.000013
Copper	0.000017
Brass	0.000018
Platinum	0.000009
Cast Iron	0.000011
Steel	0.000012
Gold	0.000015
Silver	0.000021
Zinc	0.000029
Tin	0.000025
Lead	0.000028

As these coefficients differ slightly for different specimens of the same substance, it must be understood that the numbers given in this table represent only the approximate average value.

CALCULATIONS.

23. WE shall consider calculations involving only the mean coefficient of expansion. The experimental data, for determining true coefficients of expansion, may be obtained by the methods described above; but the method of reducing these data cannot be considered here. It should be noticed that, in defining the mean coefficient of expansion, we have to consider the ratio of the mean expansion, per degree rise of temperature, to the original length or volume, as the case may be, at 0°C . Thus, if the length of a rod be L_t at $t^{\circ}\text{C}$., and $L_{t'}$ at a lower temperature $t'^{\circ}\text{C}$., then the mean expansion per degree is $\frac{L_t - L_{t'}}{(t - t')}$, and the mean *coefficient* of linear expansion (l), between

t° and t'° , is given by $l = \frac{L_t - L_{t'}}{L_o(t - t')}$, where L_o is the length of the rod

at 0°C . In working examples, in addition to the simple relation, $L_t = L_o(1 + lt)$, it is useful to have a relation between L_t and $L_{t'}$. This relation is easily obtained thus:—

$$\frac{L_{t'}}{L_t} = \frac{L_o(1 + lt')}{L_o(1 + lt)} = \frac{1 + lt'}{1 + lt}. \quad (\text{Art. 16.})$$

Or—

$$L_{t'} = L_t \frac{1 + lt'}{1 + lt}. \quad (1)$$

Since l is a very small quantity, we may, if t and t' are not very great, consider lt and lt' as small quantities, and apply the approximation of Art. 14. This gives—

$$L_{t'} = L_t(1 + lt' - lt).$$

That is—

$$L_{t'} = L_t[1 + l(t' - t)]. \quad (2)$$

This establishes a relation between $L_{t'}$, L_t , and l , which is very useful in calculating any one of these quantities, when the other two are given.

Similarly, in addition to $V_t = V_o(1 + ct)$, we have—

$$V_{t'} = V_t[1 + c(t' - t)]. \quad (3)$$

It is very important to remember that these are only approximate

relations, which are nearly true only when l and c are very small. They should therefore be applied only to the expansion of solids.

In the case of liquids, the application of the relation of formula (3) gives a rough approximation, which is sufficiently accurate for most purposes. For accuracy, a formula corresponding to (1) should be employed.

In the case of gases the expansion is far too great to admit of these approximations.

EXAMPLES III.

1. The length of an iron rod at 0°C . is 100 cm. Find its length at 10°C ., the mean coefficient of linear expansion of iron being $\cdot 000012$.

Here—

$$\begin{aligned} L_t &= L_o(1 + lt) \\ \therefore L_t &= 100[1 + (\cdot 000012 \times 10)] \\ &= 100[1\cdot 00012] = 100\cdot 012 \text{ cm.} \end{aligned}$$

2. The volume of a piece of glass at 100°C . is 100·258 c.cm., and its volume at 0°C . is 100 c.cm. Find the mean coefficient of cubical expansion of glass between 0°C . and 100°C ., and thence deduce approximately the mean coefficient of linear expansion between the same limits of temperature.

$$V_t = V_o(1 + ct).$$

Or—

$$c = \frac{V_t - V_o}{V_o t}.$$

Here then—

$$c = \frac{100\cdot 258 - 100}{100 \times 100} = \frac{\cdot 258}{10000} = \cdot 0000258.$$

We have seen that, when l is small, $c = 3l$ or $l = \frac{c}{3}$. Hence, the mean coefficient of linear expansion, as required, is—

$$\frac{\cdot 0000258}{3} = 0\cdot 0000086.$$

3. The length of a copper rod at 10° is 200·034 cm. Find its length at 100°C ., the mean coefficient of linear expansion of copper being $\cdot 000017$.

Applying formula (2) to data of the question, we have—

$$L_{100} = L_{10}(1 + 90l).$$

That is—

$$\begin{aligned} L_{100} &= 200\cdot 034[1 + (90 \times \cdot 000017)] \text{ cm.} \\ &= 200\cdot 034(1\cdot 00153) \text{ cm.} \\ &= 200\cdot 34005 \text{ cm. (approximately).} \end{aligned}$$

The accurate length at 100° found by employing formula (1) is 200·34 cm.

7. Show that the mean coefficient of superficial expansion for a given substance is approximately equal to twice the mean coefficient of linear expansion for the same substance.

8. A brass and a steel rod are each one metre long at 10°C . ; find the difference in their length at 60°C .

9. A platinum wire is found to be 0.013 cm. longer at 60°C . than at 40°C . Find the length of the wire at 0°C . Why can a platinum wire be easily fused into a glass tube ?

10. The volume of a mass of lead at 50°C . is 50 c.cm., and at 80°C . its volume is found to be 50.126 c.cm. Show that the mean coefficient of cubical expansion of lead between 50°C . and 80°C . is approximately 0.000084.

24. Change of Density with Temperature. We have seen that, in general, when a body is heated it expands—that is, its volume increases—and, since the mass of the body remains constant, it must necessarily follow that its density decreases. For, if V_0 denote the volume at 0°C . and V_t the volume at t° , also if d_0 denote the density at 0°C . and d_t the density at $t^{\circ}\text{C}$., then, since the mass remains constant—

$$V_0 d_0 = V_t d_t.$$

That is—

$$\frac{d_0}{d_t} = \frac{V_t}{V_0}.$$

But we know that $V_t = V_0 (1 + ct)$, where c denotes the coefficient of cubical expansion.

Therefore—

$$\frac{d_0}{d_t} = \frac{V_t}{V_0} = \frac{V_0 (1 + ct)}{V_0} = 1 + ct,$$

Or—

$$d_t = \frac{d_0}{1 + ct}. \quad (1)$$

This is true, as it stands, for solids, liquids, and gases ; but for solids and some liquids we may have, when t is small enough, an approximate formula, giving—

$$d_t = \frac{d_0}{1 + ct} = d_0 \frac{1}{1 + ct} = d_0 (1 - ct). \quad [\text{Art. 14 (2).}]$$

Also corresponding to formula (3), above, we have—

$$\frac{d_t}{d_{t'}} = \frac{V_{t'}}{V_t} = \frac{V_0 (1 + ct')}{V_0 (1 + ct)} = \frac{1 + ct'}{1 + ct} = [1 + c(t' - t)]. \quad (2)$$

EXAMPLES IV.

1. The density of a piece of glass at 10° C. is 2.6, and at 60° C. it is 2.5966. Find the mean coefficient of cubical expansion of glass.

Applying—

$$\frac{d_t}{d_{t'}} = [1 + c(t' - t)],$$

$$\frac{2.6}{2.5966} = 1 + 50c,$$

$$1.00131 = 1 + 50c,$$

Or—

$$c = \frac{.00131}{50} = .0000262.$$

2. The density of mercury at 0° C. is 13.596. Find its density at 100° C., the mean coefficient of cubical expansion of mercury between 0° C. and 100° C. being .000181.

From—

$$d_t = \frac{d_o}{1 + ct}, \text{ we have—}$$

$$d_t = \frac{13.596}{1.0181} = 13.354.$$

From the approximate relation—

$$d_t = d (1 - ct),$$

$$d_t = 13.596(1 - .0181)$$

$$= 13.596(.9819)$$

$$= 13.350,$$

which is a fairly close approximation.

4. Compare the density of lead at 100° C. with its density at -100° C., assuming its coefficient of expansion to remain constant within these limits of temperature.

6. Find the mass of a cubic centimetre of silver at 250° C., the density of silver at 0° C. being 10.31 grams per c.cm.

7. The density of water at 0° C. is 0.999871, and at 4° C. it is 1. Find the mean coefficient of expansion of water between 4° C. and 0° C.

CHAPTER IV.

EXPANSION OF LIQUIDS.

25. Apparent and real expansion. In dealing with the expansion of liquids, the problem becomes complicated by the expansion of the containing vessel. If, as is generally the case, the liquid expand more than the vessel, then the former *appears* to expand; but this *apparent* expansion will be less than the *real* or *absolute* expansion of the liquid, because the containing vessel has increased in volume, and the liquid has not only shown a visible increase in volume, but has also filled up this increased volume of the vessel.^e To consider the matter more definitely, let us suppose that we are noticing the expansion of mercury, in a graduated glass vessel, the graduations of which are correct at 0°C . First, imagine the vessel, and the contained mercury, to be at 0°C .; then, since the graduations are correct at this temperature, the volume of the mercury, as indicated by the vessel, is its true volume. Let this be denoted by V_0 . Next, suppose the mercury, and the vessel, to be at a temperature $t^{\circ}\text{C}$. The volume of the mercury, *as indicated by the graduations of the glass vessel*, will not now be its true volume; for, the vessel having expanded, the volume between each graduation has increased, and hence the *apparent* volume of the mercury is less than its true volume. Let V_a denote this apparent volume, then the reading V_a , although made at $t^{\circ}\text{C}$., means that the portion of the vessel up to the graduation marked V_a , had, at 0°C ., a true volume V_a . We must therefore find what a volume V_a at 0°C . becomes at $t^{\circ}\text{C}$. If c denote the mean coefficient of expansion of the glass vessel, and V_t the required volume at $t^{\circ}\text{C}$., we have—

$$V_t = V_a(1 + ct). \quad (1)$$

That is, $V_a(1 + ct)$ is the true volume of the mercury at

t° C. Now, V_0 is the real volume of the mercury at 0° C., and $V_a(1+ct)$ or V_t is its real volume at t° C.; hence, if c_r denote the mean coefficient of real expansion of mercury, we have—

$$V_t = V_0(1 + c_r t).$$

Therefore, substituting for V_t from (1)—

$$V_a(1 + ct) = V_0(1 + c_r t). \quad (2)$$

Next, consider the *apparent* expansion of the mercury. Its volume at 0° C. is V_0 , and its *apparent* volume at t° is V_a ; hence, if c_a denote the mean coefficient of *apparent* expansion of mercury, in a glass vessel, then—

$$V_a = V_0(1 + c_a t). \quad (3)$$

Now, from (2)—

$$\frac{V_a}{V_0} = \frac{1 + c_r t}{1 + ct},$$

and from (3)—

$$\frac{V_a}{V_0} = 1 + c_a t.$$

Therefore—

$$\frac{1 + c_r t}{1 + ct} = 1 + c_a t.$$

Or—

$$1 + c_r t = (1 + c_a t)(1 + ct). \quad (4)$$

This gives us an accurate relation between c_r , c_a , and c ; but, since these quantities are small compared with unity, we may by Art. 14 (2) obtain a more convenient approximate relation, thus—

$$(1 + c_r t) = (1 + c_a t + ct) = [1 + (c_a + c)t].$$

Or—

$$c_r = c_a + c. \quad (5)$$

That is, the mean coefficient of *real* expansion of a liquid is approximately equal to the sum of the mean coefficient of *apparent* expansion of the liquid in any vessel, and the mean coefficient of expansion of that vessel.

What we have said above applies directly to cubical expansion, but it is also applicable to linear expansion.

For, suppose that we are measuring the length of a brass rod with a steel scale, the graduations on which are correct at 0°C. , then, if L_o denote the true length of the rod at 0°C. , L_a its *apparent* length at $t^\circ \text{C.}$, and L_t its *true* length at $t^\circ \text{C.}$, we have, in exactly the same way as explained above—

$$L_t = L_a (1 + lt) \quad (1')$$

$$L_a (1 + lt) = L_o (1 + l_r t) \quad (2')$$

$$L_a = L_o (1 + l_a t) \quad (3')$$

$$1 + l_r t = (1 + l_a t) (1 + lt) \quad (4')$$

$$l_r = l_a + l. \quad (5')$$

These correspond respectively to 1, 2, 3, 4, 5, above :

l_r being the mean coefficient of real linear expansion of the brass rod.

l_a	"	"	apparent	"	"	"
l	"	"	linear expansion of the steel scale.			

From this, then, it appears that, before we can find the real expansion of any liquid, we must know, accurately, the expansion of some solid such as glass, and then measure the apparent expansion of the liquid in a glass vessel. On applying the relation expressed in (5), we should then be able to deduce c_r from c_a and c . It is, however, difficult to measure the mean coefficient of expansion of any solid directly with any great accuracy. Moreover, the expansion of different specimens of the same substance vary greatly with composition and previous treatment; hence, it would not be sufficient to know the expansion of the material composing the vessel, but the expansion of the actual piece of material forming the vessel, after it has been formed into that vessel, must be known. This further adds to the difficulty. There is, however, another way of considering the matter. If the real expansion of a liquid can be determined, we may, by measuring the apparent expansion of this liquid in a vessel, determine [from (5)] the expansion of the material of that vessel, which may then be used to determine the real expansion of other liquids. The measurement of expansion in this way involves the determination, by some absolute method, of the real

expansion of a given liquid, and this determination is then made the starting-point of all other expansion measurements. This is the method which has been most generally adopted. First, Dulong and Petit, and afterwards Regnault, measured, with great accuracy, the absolute expansion of mercury, and, as mercury can readily be obtained in a fairly pure state, the result of their experiments gives us a definite starting-point for other measurements.

26. Determination of the absolute expansion of mercury.

The methods adopted, for the determination of this constant, by Dulong and Petit and by Regnault, were both based on the same principle, and differed only in experimental detail.

The method of Dulong and Petit. The principle of this method is simple. If we have two vertical tubes, communicating by a horizontal branch, and each contain a liquid of given density, then, for equilibrium, the pressures due to the liquid columns on each side of a section, taken anywhere in the horizontal branch, must be equal. Thus, consider the pressures on the section ab in Fig. 23. Let h and h' denote the depth of the centre of gravity of the section below the surfaces of the liquid in T and T' respectively. Then, if a denote the area of the cross section ab , we have, from hydrostatical principles, that the pressure on it due to the column in $T = h d a g$, and that due to the column in $T' = h' d' a g$, where d and d' are the densities of the liquids in T and T' respectively, and g the acceleration due to gravity. But, since ab is in equilibrium, we must have

$$h d a g = h' d' a g.$$

That is—

$$h d = h' d'.$$

It is evident that this is true whatever value a may have, and whatever be the shape and size of T and T' .

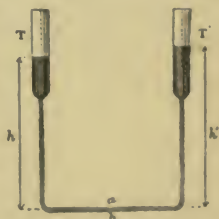


Fig. 23.

If, now, the tubes contain mercury, and one column, T, be kept at 0° C., while the other, T', is raised to a temperature t° C., we have—

$$h_o d_o = h_t d_t.$$

$$\therefore \frac{h_t}{h_o} = \frac{d_o}{d_t} = 1 + ct. \quad (\text{Art. 24.})$$

$$\therefore \frac{h_t}{h_o} - 1 = ct.$$

Or—

$$c = \frac{h_t - h_o}{h_o t}.$$

Thus, the mean coefficient of absolute cubical expansion can be determined by noting $(h_t - h_o)$, h_o , and t , and the result is quite independent of the expansion of the tubes containing the mercury.

The apparatus used by Dulong and Petit is shown in Fig. 24. The tube T' was enclosed in a cylinder, B, and

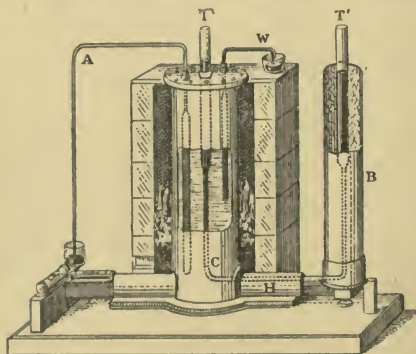


Fig. 24.

surrounded with melting ice. T was placed in a thick copper cylinder, C, which was filled with oil; and, for the purpose of heating it, a furnace was built round it. This is shown in section in the figure. The temperature of this oil bath was given by two thermometers—a mercury weight thermometer, W, and an air thermometer, A. These two

kinds of thermometers will be described later on; it is, however, interesting to notice that this is the first time air was used as a thermometric substance to the exclusion of mercury. Dulong and Petit remarked that, at high temperatures, the indications of the two instruments did not agree; and so they decided to base their calculations on the indications of the air thermometer.*

The heights of the columns of mercury were read by means of the cathetometer,† which was invented expressly for this purpose. The level of the mercury in T was first read, then the level in T', and finally the level of the *axis* of the horizontal tube, H. The difference of the first two readings on the cathetometer scale gave $h_t - h_o$, and the difference of the last two readings gave h_o . The temperature $t^\circ \text{C.}$ of the oil bath, and therefore of the mercury in the tube T, was given by the air thermometer A. Thus, all the data for calculating the mean coefficient of absolute cubical expansion of mercury were determined.

The chief results obtained by Dulong and Petit were:—
The mean coefficient of absolute expansion of mercury

$$\text{between } 0^\circ \text{ and } 100^\circ = \frac{1}{5500};$$

$$,, \quad 0^\circ \text{ and } 200^\circ = \frac{1}{5425};$$

$$,, \quad 0^\circ \text{ and } 300^\circ = \frac{1}{5300}.$$

* It is important to notice that if Dulong and Petit had used only a *mercury* thermometer, they would not have detected any irregularity in the expansion of *mercury*.

† This instrument consists essentially of a vertical brass or steel scale, to which is attached a horizontally fixed telescope, capable of motion up and down the scale. To determine the vertical distance between any two points, the telescope is focussed on the higher point, and its position on the vertical scale read off. It is then lowered and focussed on the lower point, and the corresponding scale reading again taken. The difference of the two readings thus obtained gives the required vertical distance between the points. The telescope is furnished, in its field of view, with two fine cross wires at right angles to each other, and, in focussing, its position on the scale is so adjusted that the point viewed is made to coincide with the intersection of these cross wires.

This shows that the expansion of mercury increases as the temperature rises.

27. Absolute expansion of liquids. Having now determined the absolute expansion of mercury, we are able to determine the absolute expansion of any given liquid in two general ways.

(1) **The weight thermometer method.** The weight thermometer is usually a glass tube of the form shown in Fig. 26. The bulb is large, and generally of a cylindrical form; the stem, like that of the mercurial thermometer, is of thick capillary tubing with a fine bore. When used to determine the absolute expansion of a liquid, the experiment consists of two separate steps.



Fig. 26.

(a) The determination of the apparent expansion of the mercury in the tube, and then, the real expansion of mercury being known, we can deduce the expansion of the thermometer tube by the help of the relation of Art. 25, formula (5), where $c_r = c_a + c$, or $c = c_r - c_a$. Here, c_r is obtained from Regnault's results, c_a is observed, and c is determined by calculation.

(b) The expansion of the tube being thus determined, we may measure the apparent expansion of any liquid in it, and then we can, as above, deduce the real expansion from the relation $c_r = c_a + c$. Here, c_a is observed, c is known from (a) above, and thus c_r is determined.

This method of determining the expansion of *any* liquid, when the real expansion of mercury is known, is quite general and very accurate. We shall therefore consider, more fully, the method of experiment and calculation employed in steps (a) and (b) above. In both cases it is required to determine the apparent expansion of a liquid in the weight thermometer tube, and, as this process is the same for any liquid, we shall describe the method of determining the *apparent* expansion of mercury, between 0°C. and 100°C. , by means of the weight thermometer. The tube, which should be perfectly clean and dry, is weighed, and the weight noted. Let it be w . It is then carefully filled with

mercury, by alternately warming the bulb and dipping the end of the stem under mercury, and finally, boiling the mercury, in exactly the same way as that adopted in filling the mercurial thermometer (Art. 9). When quite full, a small porcelain capsule containing mercury is placed so that the end of the thermometer stem dips under the surface of the mercury, as shown in Figs. 24 and 26, and the whole is allowed to cool. As the mercury in the tube cools, it contracts and mercury from the capsule passes into the tube, so that so long as the end of the stem is kept below the surface of the mercury no air gets in, and the tube is kept full of mercury. When sufficiently cool, the thermometer is placed in a suitable vessel, and surrounded with melting ice, the capsule being still carefully kept in position. Further contraction of the mercury in the tube takes place, more mercury passes in, and finally the tube is filled with mercury at 0°C . The capsule is then withdrawn and replaced by another which has previously been carefully cleaned, dried and weighed. Let its weight be denoted by x . The thermometer is now taken out of the ice, the capsule being held under the end of the stem; as the mercury takes the temperature of the room it expands, and a small quantity overflows into the capsule. When this overflow has quite ceased, the tube and capsule are again weighed. Let their weights be W and X respectively. Then $(W-w)$ denotes the weight of mercury in the thermometer, and $(X-x)$ the weight of mercury in the capsule, and $(W-w)+(X-x)$ gives the total weight of mercury in the thermometer at 0°C . Let this be denoted by W_0 . The thermometer is now placed in a steam chamber (Fig. 9), and the capsule, and bent portion of the stem, arranged outside as indicated at W , Fig. 24. As the mercury expands, it overflows into the capsule, and after some time the expansion is complete and the overflow ceases. The capsule is now taken and again weighed. Let its weight be X' . Then $(X'-x)$ denotes the weight of mercury expelled from the thermometer owing to the expansion between 0°C . and $T^{\circ}\text{C}$, where $T^{\circ}\text{C}$. is the temperature of the steam. Let $(X'-x)$ be denoted by w' . Then $(W_0 - w')$ is the quantity

of mercury which fills the thermometer at $T^{\circ}\text{C}$. We have now obtained all the data for calculating the mean coefficient of *apparent* expansion of mercury, in the thermometer, between 0°C . and $T^{\circ}\text{C}$.

If we notice the thermometer after the experiment is over, we shall see, that as the mercury cools it contracts, and, after it has cooled down to 0°C ., a vacant space, which was originally occupied by the mercury which has been expelled, will be left in the upper part of the tube. Hence, if *the mercury remaining in the tube* be now supposed to expand, the apparent expansion, for a rise of temperature from 0°C . to $T^{\circ}\text{C}$., will be denoted by the volume of this vacant space, and the ratio of the apparent expansion to the original volume at 0°C . will be denoted by $\frac{w'}{W_o - w'}$, for w' is the weight of mercury filling the vacant space at 0°C ., and $(W_o - w')$ the weight of mercury remaining in the tube, and the volumes occupied, at 0°C ., by these weights are proportional to the weights themselves. Hence, the mean coefficient of apparent expansion between 0° and T° is given by—

$$c_a = \frac{w'}{(W_o - w')T} \quad (1)$$

where W_o is the weight of mercury in the thermometer at 0°C . and w' the weight of mercury which overflows between 0°C . and $T^{\circ}\text{C}$. If c_a be considered known, then, the above data, and the relation between them expressed by (1), may be employed for the determination of temperature. For, from (1), we have—

$$T = \frac{w'}{(W_o - w')c_a} \quad (2)$$

Thus, the process described above might have been employed to determine T .

The weight thermometer can thus be used as a true thermometer; as such, it has the great disadvantage that a difficult experiment is necessary for the determination of each individual temperature. It may be conveniently used as a maximum thermometer, for the value of T given by (2) always represents the maximum temperature experienced.

(2) **Calibration method.** In this method a tube like a large thermometer tube is employed. It is first calibrated in such a way that each division on the stem is of equal volume, and the volume of the bulb is known in terms of the divisions on the stem—that is, if a division on the stem be marked n , it means that the volume of the bulb and of the stem up to that point is n times the volume of one division.

This tube is then filled with mercury at 0°C. up to a certain division on the stem. It is then raised to a temperature T in a water, steam, or oil bath, and the apparent expansion of the mercury, as indicated by the position of the mercury column in the stem, is noted. Let n be the reading on the stem at 0°C. and n' the reading at $T^{\circ}\text{C.}$, then for the mean coefficient of apparent expansion of mercury we have $c_a = \frac{n' - n}{nT}$. This is perhaps the simplest

and most accurate method of studying liquid expansion, but the preliminary calibration of the tube is a rather difficult process. When c_a for mercury is thus determined, we can, as in the preceding method, obtain c from the relation $c_r = c_a + c$, and the value of c_r for any given liquid can be obtained by measuring c_a and again applying this relation.

M. Pierre has employed this method to determine the expansion of a large number of liquids.

29. Expansion of water. The expansion of water has been the subject of numerous experiments. It is of special interest, not only because water is so generally used in physical operations, but also because it presents noteworthy peculiarities. Amongst others, Pierre, Despretz, Hällstrom, and Mathiessen have studied this question: the last two adopted a method essentially similar to that described in Art. 28; the first two employed method (2) of Art. 27, which was found to have this advantage, that the water in the tube could be cooled down to between 10° and 20° below zero without freezing, and in this way the expansion of *water* at temperatures below the freezing point was studied. As the result of experiment, it was found that as water cooled down from, say 10°C. to -10°C. , it passed through

a point of maximum density at about 4°C. —that is, as it cooled it contracted, and consequently its density increased until 4°C. was reached; then it began to expand, and continued to do so as far as the observations were carried. Hence, if water at 0°C. be gradually heated, it contracts until a temperature very close to 4°C. is reached; it then begins to expand, and continues to do so as the temperature rises. This can be shown by method (2) of Art. 27. The tube and part of the stem are filled with water and surrounded with melting ice. The reading on the stem is noted, the ice removed, and the apparatus allowed to gradually take the temperature of the room. As the temperature slowly rises, the level of the water column in the stem falls, showing contraction of the water. This continues until a temperature of about 6°C. is reached, and then the column of water begins to rise, showing that the point of maximum density (or minimum volume) has been passed, and that the water is now expanding. It must be remembered that we are now considering the *apparent* expansion of water in a glass tube, and that the simultaneous contraction of the water and expansion of the tube exaggerate the effect, and hence the water does not *appear* to expand until a temperature of about 6° is reached, whereas it *really* begins to expand at about 4°C. , but this is not apparent until the rate of expansion of the water becomes greater than that of the glass. From readings of the position of the water column at different temperatures, the mean coefficient of *apparent* expansion (c_a) can be determined; and if the coefficient of expansion (c) of the tube is known, then c_r is given by $c_r = c_a + c$. To find the temperature of maximum density by this method, we must find the temperature at which the *true* coefficient of real expansion (Art. 16) is zero; for, if $t^{\circ}\text{C.}$ be the temperature of maximum density, then between 0°C. and $t^{\circ}\text{C.}$ this coefficient is negative, above $t^{\circ}\text{C.}$ it is positive, and at $t^{\circ}\text{C.}$ it is zero.

A simple experiment, devised by Hope, and hence called **Hope's experiment**, illustrates these facts, and serves to determine approximately the temperature of maximum

density. A tall glass jar, having two side openings fitted with thermometers, as shown in Fig. 27, is surrounded at its centre with an outer vessel containing a freezing mixture (Art. 51). The vessel is filled with water at the ordinary temperature, and allowed to cool under the influence of the freezing mixture. As cooling goes on, the lower thermometer falls steadily, while the upper one undergoes but little change; but as the temperature of the former approaches $4^{\circ}\text{C}.$, it falls more and more slowly, and finally becomes stationary; meanwhile, the upper thermometer begins to fall more rapidly, and continues to do so until $0^{\circ}\text{C}.$ is reached. Ultimately, a thin crust of ice begins to form on the surface of the water, and the lower thermometer remains stationary at $4^{\circ}\text{C}.$ These facts admit of simple explanation. The freezing mixture cools the water nearest it, this contracts, becomes denser, and descends, causing the lower thermometer to fall rapidly. This goes on so long as a fall of temperature causes the water to contract, but below $4^{\circ}\text{C}.$ expansion sets in, and then the water cooled by the freezing mixture rises instead of falls, and causes the rapid fall of the upper thermometer to $0^{\circ}\text{C}.$, while the lower thermometer remains stationary at the temperature of the lowest, and therefore densest, layer; and its reading consequently indicates, approximately, the temperature of maximum density of water. This experiment illustrates what actually takes place in pools of water during frosty weather; the surface freezes, but the temperature of the deeper layers of water seldom falls below $4^{\circ}\text{C}.$, and thus the lives of fish and other aquatic animals are preserved. It should be noticed here that water expands on freezing, and thus the ice floats on the surface. Were this not the case, each layer of ice would sink as it was formed, and ultimately all the water in the pools would be converted into ice.



Fig. 27.

CALCULATIONS.

30. It will now be convenient to collect together the various formulæ deduced in the preceding pages.

1. Apparent and real expansion.

$$(a) \quad l_r = l_a + l. \quad \text{Art. 25 (5')}.$$

$$(b) \quad c_r = c_a + c. \quad \text{Art. 25 (5)}.$$

These two formulæ [(a) and (b)] should be learnt in words.

$$(c) \quad L_a = L_o (1 + l_a t). \quad \text{Art. 25 (3')}.$$

$$(d) \quad V_a = V_o (1 + v_a t). \quad \text{Art. 25 (3)}.$$

These formulæ establish a relation between *apparent* length or volume at $t^\circ C$. and *true* length or volume at $0^\circ C$.

2. Weight thermometer.

$$(g) \quad c_a = \frac{w'}{(W_o + w')} T. \quad \text{Art. 27 (1)}.$$

$$(h) \quad T = \frac{w'}{(W_o - w')} c_a. \quad \text{Art. 27 (2)}.$$

The reader should make himself thoroughly familiar with these formulæ before going any farther. In studying any formula the following plan should be adopted: (1) Thoroughly master the method by which the relation is obtained. (2) Note carefully the conditions under which it is applicable. (3) If the formula is fundamental, it should be learnt; but, in all cases, the method of work is of more importance than the formulated result.

The following examples illustrate the application of the above relations.

EXAMPLES V.

1. A zinc rod is measured by means of a brass scale, and found to be 1.0001 metres long at $10^\circ C$. What is the real length of the rod at $0^\circ C$. and at $10^\circ C$.? [Mean coefficient of linear expansion of zinc is .000029 and of brass .000019.]

Applying (c) above we get—

$$L_a = L_o (1 + l_a t).$$

$$\therefore L_o = \frac{L_a}{1 + l_a t} = L_a (1 - l_a t).$$

Here—

$$L_a = 1.0001 \{ l_a = (l_r - l) = (.000029 - .000019) = .00001 ; \text{ and } t = 10. \\ \therefore L_o = 1.0001 [1 - (.00001) 10] \\ = 1.0001 [1 - .0001] \\ = 1 \text{ nearly.}$$

Also—

$$L_t = L_o (1 + l_r t). \quad (\text{Art. 16.})$$

Here—

$$L_o = 1 \text{ nearly, and } l_r = 0.000029. \\ \therefore L_{10} = 1(1 + 0.00029) \\ = 1.00029.$$

Or—

$$L_t = L_a (1 + lt). \quad (c) \\ \therefore L_{10} = 1.0001(1 + .000019 \times 10) \\ = 1.0001(1 + .00019) \\ = 1.00029 \dots$$

4. An ordinary mercurial thermometer is placed with its bulb and lower part of stem in a vessel of water, and indicates a temperature T . The upper portion of the stem, containing n divisions of the mercury column, is in the air at a temperature t . What is the true temperature of the water in the vessel?

The true temperature of the water, T' , is that which the thermometer would indicate if completely immersed in the water. If this were the case, the n divisions of the mercury column, which are now at t° , would be at T° . We have therefore to find the expansion of these n divisions for a rise in temperature from t° to T° . If c_a denote the mean coefficient of apparent expansion of mercury in glass we have—

$$n' = n[1 + c_a (T' - t)].$$

But—

$$T' = (T - n) + n' \\ = T + n' - n \\ = T + n(T' - t)c_a. \quad (\text{Cp. Art. 12, 4.})$$

5. A weight thermometer weighs 50 grams when empty, and 710 grams when full of mercury at 0°C . On heating up to 100°C ., 10 grams of mercury are expelled. Calculate the mean coefficient of cubical expansion of glass, assuming the mean coefficient of real cubical expansion of mercury to be 0.000181.

Applying (*g*) above we get—

$$c_a = \frac{w'}{(W_o - w')T}$$

$$\therefore c_a = \frac{10}{(660 - 10)100} = \frac{1}{6500}$$

$$\therefore c_a = 0.000154 \text{ nearly.}$$

But—

$$c_r = c_a + c. \quad (b)$$

$$\therefore c = c_r - c_a$$

$$= 0.000181 - 0.000154$$

$$= 0.000027.$$

11. A rod of copper and a rod of iron placed side by side are riveted together at one end. The iron rod is 150 cm. long, and a mark is made on the copper rod, showing the position of the unriveted end of the iron at 0° C. If at 30° the mark is 0.0255 cm. from the end of the iron rod, what is the coefficient of expansion of copper, that of iron being 0.000012?

16. The coefficient of absolute cubical expansion of mercury is .00018, the coefficient of linear expansion of glass is .000008. Mercury is placed in a graduated glass tube, and occupies 100 divisions of the tube. Through how many degrees must the temperature be raised to cause the mercury to occupy 101.56 divisions?

17. A porcelain weight thermometer weighs 165 grams when empty and 468 grams when full of mercury at 0° C. When heated to 300° C., the weight of overflow is found to be 13.464 grams. Find the mean coefficient of cubical expansion of porcelain between 0° and 300° C., assuming that of mercury to be .000184 for the same range of temperature.

21. If δ be the expansion of water between 4° and 0° C., and Δ its expansion between 4° and t° , show what is the density of water at t° referred to water at 0°.

CHAPTER V.

EXPANSION OF GASES.

31. THE study of the expansion of gases is greatly complicated by the influence of pressure. If a solid or liquid be heated it will expand and overcome very great pressures in doing so; so that, under ordinary circumstances, the expansion of liquids and solids is quite independent of the pressure at which the expansion takes place. Moreover, the volume of a solid or liquid is, within very wide limits, independent of external pressure; for example, the volume of a block of iron or a quantity of water is practically the same under a pressure of 10 atmospheres as under a pressure of 1 atmosphere. This is not the case with gases, for we know, from Boyle's law, that the volume of a gas is inversely proportional to its pressure.

32. The relation between the volume, temperature, and pressure of a gas. Before proceeding to the experimental study of the expansion of gases it will be well to consider the question theoretically, and see exactly where, and in what direction, experiment is necessary.

Suppose we consider a definite *mass* of gas; before we can realise its state, we must know three things about it: (1) its volume; (2) its pressure; (3) its temperature. For example, if we wish to enclose this mass of gas in a vessel, so that it may be in a particular state, we must know how large the vessel must be, its ability to resist pressure, and the temperature of its interior. The *state* of a given mass of gas thus appears to be dependent on three variables—volume, pressure, and temperature—these may be denoted by v , p , and t . It is, however, possible that some relation exists between these three variables, which may enable us to determine one of them if the other two are given. To determine if this be the case we must resort to experiment, and this question is a good example of how experimental research is conducted.

We have three variables, and we wish to determine how they are related in their effect on the state of a given mass of gas. It is evident that, if the three are allowed to vary at once, it will be impossible to determine the cause of any observed effect. For example, if p , v , and t change simultaneously, the change in v may be related either to that of p or t or of both, so that such an experiment would give no result. We must therefore allow only two of the variables to change simultaneously while the other remains constant and consequently has no part in producing the observed effects. This method of work necessitates three different sets of experiments to establish:—

(i) The relation between p and v with t constant.

(ii) " " " v " t " p "

(iii) " " " p " t " v "

We shall consider these separately.

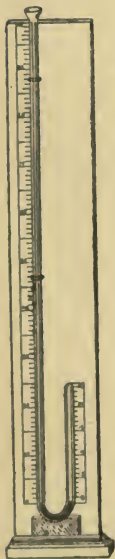


Fig. 29.

(i) **The relation between pressure (p) and volume (v) at constant temperature (t).** To determine this relation we have evidently to maintain a definite, constant mass of gas at constant temperature, and investigate how change of pressure affects the volume occupied by this mass of gas. By a number of experiments Boyle and Mariotte established the law that, under these circumstances, the volume varies inversely as the pressure: for example, if the pressure is doubled the volume is halved. This law may be verified by means of the bent tube (sometimes called Boyle's tube) shown in Fig. 29. The gas to be experimented on is placed in the shorter closed limb, and the quantity of mercury in the bend is adjusted so that the level, in each limb, is at the zero of the corresponding scale and also in the same horizontal line. Mercury is then poured into the long limb, and it is found that as the pressure on the gas in the closed tube is increased (Art. 14, 7) its volume diminishes, and by observing the volume corresponding to

different pressures the law given above will be found true. The result of experiment for this relation may be expressed by writing—

$$p \propto \frac{1}{v}; \text{ i.e., } p = \frac{k}{v};$$

or—

$$p v = k, \quad (i)$$

where k is a constant. This is known as **Boyle's Law**.

(ii) **The relation between volume (v) and temperature (t) when pressure (p) is constant.** This is the relation corresponding to the one already considered in the case of solids and liquids. We have to determine how the volume of a given mass of gas changes under the influence of temperature; it is found that it increases with rise of temperature just as does that of solids and liquids, but the mean coefficient of expansion, for a given range of temperature, is much greater. The two most important experimental determinations of this relation were carried out by Gay Lussac and Regnault. The observations of the latter were by far the more accurate, but as the results obtained by the former were considered correct for a long period of time, we shall briefly consider both methods.

1. *Gay Lussac's method.* For the purpose of this determination Gay Lussac obtained a large thermometer tube with a spherical reservoir and a long stem. This tube he carefully calibrated and graduated after the plan mentioned in Art. 27, 2. It was then filled with mercury and fitted (as at A, Fig. 30) into a wider tube, T, filled with calcium chloride. By introducing a platinum wire, w, into the tube the mercury was

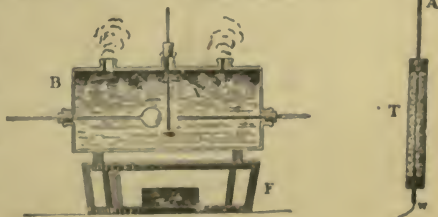


Fig. 30.

slowly shaken out, and its place taken by air which was quite dry owing to it having passed over the calcium chloride in the wide tube. A short column of mercury

was left in the stem of the tube to act as an index; it was then taken and placed horizontally (as at B, Fig. 30) in a sheet-iron vessel, V, which was first filled with melting ice and afterwards placed on a furnace, F, and gradually heated. The position of the mercury index was noted when the bulb of the tube was surrounded with ice, and also at different temperatures as the heating went on. From these observations the mean coefficient of *apparent* expansion of air in glass was calculated and the mean coefficient of *absolute* expansion deduced, from this result and the known mean coefficient of expansion of the glass tube. It will be seen that this method is identical in principle with that of Art. 27, 2. In this way Gay Lussac found $c_r = 0.00375$. This result we now know to be too high.

2. *Regnault's method.* In Gay Lussac's method, as described above, there are two important sources of error.

(a) The moisture clinging to the interior of the thermometer tube was not removed, and, as it was converted into vapour on heating, the volume of the enclosed air appeared to increase more than it really did, hence the result obtained was too high. (b) The index of mercury in the stem did not completely separate the enclosed air from the outer air. Regnault arranged his apparatus so as to avoid these errors; the interior of the vessel A (Fig. 31) which contained the gas was thoroughly dried by alternately filling it with dry

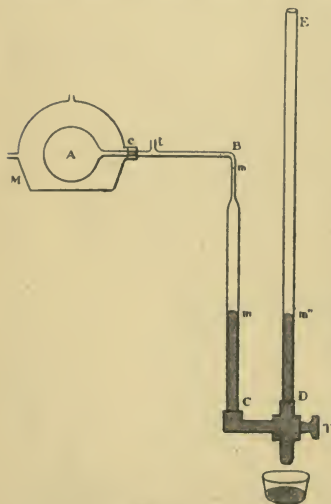


Fig. 31.

warm gas and exhausting it, through the tube *t*, by means of an air-pump. The mercury index in the stem was done

away with, and its place taken by the mercury column in a very convenient manometric arrangement with which A was connected. The details of Regnault's method, both as regards apparatus and calculation of results, are too complicated to be entered into here; we may, however, notice it sufficiently to indicate the principle of the method and the main features of the apparatus. The essential parts of the apparatus used by Regnault are shown in Fig. 31. It consists of two parts: (1) the reservoir A in which the gas is placed, and (2) the manometer B C D E. The reservoir is a small spherical vessel which can be enclosed in a suitable metal vessel, M, and the temperature of the contained gas adjusted by filling the space between the two, with ice, water, steam, etc. The manometer consists essentially of two vertical tubes communicating below by a metal bend to which a tap is attached. One of the tubes, B C, ends in a fine piece of tubing which is bent over at B and joined to the stem of A; the other, D E, is much higher and of uniform bore throughout. Communicating with the horizontal part between B and A is a small tube *t*, by means of which the interior of A is dried, and the gas introduced into it after passing through a series of drying tubes. To perform the experiment the reservoir, A, is surrounded with ice, and mercury is poured into the manometer at E until it rises to the mark *m* on the tube B C, and to a point in D E on the same horizontal line as *m*. This adjustment of levels is readily effected by varying, through *t*, the quantity of air in A. The tube *t* is now sealed off in the blow-pipe, the ice in M removed, and a small quantity of water put in its place and heated till it boils. The reservoir A is thus surrounded with steam at the temperature of the boiling point; the contained gas expands, and, depressing the column of mercury in B C, causes that in D E to rise. After a short time the levels become stationary; the tap T is then opened and the mercury runs out, the level in D E falling more rapidly than that in B C. By closing the tap at the right moment the levels of the mercury columns in B C and D E may be adjusted in the same horizontal line in a position similar

to that shown in the figure at m' , m'' . The air in A has thus expanded, between 0°C. and $t^\circ \text{C.}$, at a constant pressure equal to that of the atmosphere at the time of the experiment, for, both at 0°C. and $t^\circ \text{C.}$, the mercury columns in the manometer have been adjusted so as to have no difference of level. If, then, we know the relative volumes of A, of its stem up to m and of the portion of B C between m and m' , we can calculate the mean coefficient of absolute expansion in the usual way. It must be noticed here that the gas in the part of the apparatus outside M is at a different temperature to that in A, hence a correction similar to that of Ex. 4, Art. 30, must be applied in working out the result. Theoretically, c_r is obtained as in case of liquids from c_a and c , and thus c should be known; but the value of c for glass is so small, relative to c_r for gases, that it is not necessary, except when great accuracy is desired, to know its value for the vessel actually employed in the experiments; it is sufficient to know the average value for the material of the vessel. Regnault found by this method the value 0.0036706 for the mean coefficient of absolute expansion of air between 0°C. and 100°C. ; for different gases the value of this coefficient varied slightly, especially in the case of gases which are easily liquefied. Practically, however, this result, which is approximately equal to $\frac{1}{273}$, may be considered as applicable to most gases. Hence we may state that, at constant pressure, the relation between v and t is such that, *if a given volume of gas be raised, at constant pressure, from 0°C. to 1°C. it will increase in volume by approximately $\frac{1}{273}$ of its original volume at 0°C.* This is known as **Charles' Law**—sometimes as **Gay Lussac's Law**. It may be expressed by writing—

$$v_t = v_0 (1 + c_r t) \quad (\text{ii})$$

where—

$$c_r = \frac{1}{273}.$$

Also—

$$\begin{aligned} v_t &= v_0 (1 + c_r t). \\ \therefore \frac{v_t}{v_{t'}} &= \frac{1 + c_r t}{1 + c_r t'}. \end{aligned} \quad (\text{ii } a)$$

It had been stated by Faraday previous to Regnault's experiments that the value of c_p was constant, whatever the pressure at which expansion took place. Regnault investigated this, and found it to be only approximately true; the value of the coefficient increases slightly with the pressure.

(iii) The relation between pressure (p) and temperature (t) when the volume (v) remains constant. The point to be determined here is the variation with the temperature of the pressure of a given mass of gas kept at constant volume. This question was investigated by Regnault by means of the apparatus shown in Fig. 31. The method of experiment is very similar to that described in (ii); the reservoir is filled with gas, surrounded with ice, and the levels of the mercury adjusted to m in exactly the same way as described on page 59. The reservoir is next heated by steam, and, as before, the gas expands, depresses the mercury column in BC, and raises that in DE. But it is required to find the change of *pressure* with temperature, at *constant* volume, that is, when no expansion is allowed to take place. It is therefore necessary to bring the mercury column back to m , in order that the gas in A may remain at constant volume.* This can be done by pouring mercury in at E. The height of the column DE will increase, and the pressure due to the difference in height of the columns in BC and DE (Art. 8) will cause the volume of the gas in A to diminish; by sufficiently increasing this difference, the volume may be reduced to its original volume at 0° , and the level of the mercury in BC brought back to m . It thus appears that the pressure increases as the temperature rises, and if, when this constant volume adjustment is made, the difference of level between the columns of mercury in BC and DE be noted for a given rise of temperature, the mean coefficient of increase of *pressure* at constant *volume* can be calculated in the same way† as the mean coefficient of increase of *volume*

* This is not quite accurate, for the volume of A will be greater when heated than at 0°C. , owing to the expansion of the glass, and in exact calculation this change of volume has to be taken into account.

at constant *pressure*. For example, if π denote the pressure due to the observed difference of level for a rise of temperature from 0°C. to $t^\circ \text{C.}$, then $c_p = \frac{\pi}{pt}$ where p is the initial pressure of the gas at 0°C. , as given by the barometer reading at the time of experiment. As the result of his experiments, Regnault found the value of c_p for air to be 0.003665, which is very nearly equal to the value found for c_v . The value of this coefficient for different gases was found to vary in exactly the same way as that of c_v , and it was also found that in all cases, except that of hydrogen, c_p was slightly smaller than c_v . These variations are due to the fact that no gas obeys Boyle's law exactly; and they can, in fact, be predicted without experiment, if the nature of the departure from Boyle's law for any given gas is known. For ordinary purposes, however, these discrepancies are quite negligible, and we may state that the mean coefficient of increase of pressure at constant volume is equal to that of increase of volume at constant pressure, the approximate value for both being $\frac{1}{273}$. Hence the relation between p and t when v is constant is such that, *if the temperature of a given mass of gas at constant volume be raised from 0°C. to 1°C. , its pressure will increase by $\frac{1}{273}$ of its original pressure at 0°C.* This relation may be expressed by writing—

$$p_t = p_0 (1 + c_p t) \quad (\text{iii})$$

where—

$$c_p = \frac{1}{273};$$

and as in (ii)—

$$\frac{p_t}{p_{t'}} = \frac{1 + c_p t}{1 + c_p t'}. \quad (\text{iii } a)$$

The coefficient c_p is sometimes called the mean coefficient of expansion at constant volume, which is a contradiction in terms, and should not be used.

34. The air thermometer. One of the most important applications of the expansion of gases is the use of air as a thermometric substance. We have seen that gases expand much more than either solids or liquids. Hence a gas thermometer is much more sensitive than a mercurial one, the expansion of the gas for 1° C. being more than twenty times that of mercury. Experiment has also shown that the change of volume of air, with change of temperature, is perfectly uniform; and this, we have seen (Art. 26), is not the case with mercury. Further, the expansion of air is so great compared with that of the vessel which contains it, that it is necessary to know only the approximate law of expansion of the latter. There are other important reasons for the adoption of air, or other gas which closely follows Boyle's law, as a thermometric substance; these we shall consider later (Art. 43), when the question of specific heat has been dealt with. Theoretically, the air thermometer may take one of three forms; any apparatus, by means of which the mean coefficient of expansion of air at constant pressure, or the mean coefficient of increase of pressure at constant volume has been studied, may be used as an air thermometer; for if c_p be known we can use the relation, $V_t = V_o (1 + c_p t)$, to determine t , thus—

$$t = \frac{V_t - V_o}{V_o c_p}.$$

Similarly from (iii) we have—

$$t = \frac{P_t - P_o}{P_o c_p}.$$

We may therefore use as an air thermometer either of the forms of apparatus described in Art. 32. In practice, however, only one form is used. Although the great expansion of gas is in one way an advantage, it is a great inconvenience if we wish to measure temperature by expansion at constant pressure, for the expansion is so great that a comparatively large proportion of the gas necessarily occupies, on expansion, portions of the apparatus which are not at the temperature it is desired to measure. For this reason it has been found most convenient to employ

the apparatus used to determine the mean coefficient of increase of pressure at constant volume (Fig. 31, Art. 32). In this apparatus the air is maintained at constant volume, and can therefore be kept, throughout the experiment, at the temperature it is required to measure, and which is given by $t = \frac{P_t - P_o}{P_o c_p}$.

Here, P_t is known from the barometric height (H) at the time of the experiment, and the difference of level (p) of the mercury columns in the two tubes. It is equal to $(H \pm p)$. P_o is determined once for all by surrounding the bulb with melting ice and determining H and p in the same way as for P_t above; c_p is known from the results of Regnault's experiments, or may be determined by a preliminary experiment.

Owing to the fact that the properties of air remain unchanged through a very wide range of temperature, the air thermometer may be used to determine very high or very low temperatures. When used at very high temperatures the bulb is made of porcelain, and in this form it is the most reliable pyrometer or instrument for measuring high temperatures. The details of construction of air thermometers vary greatly according to the purpose for which they are required; the bulb is often very long and cylindrical, as shown at A A, Fig. 25. For other purposes it is comparatively small and spherical; but in all cases the method of experiment and the details of calculation are identical. Fig. 24, A, shows an air thermometer adjusted for use.

35. Absolute temperature. Imagine an air thermometer constructed out of a long tube of uniform bore in such a way that temperature is indicated by increase of volume at constant pressure, and let the air in the tube occupy 273 cm. of its length at 0°C . If this thermometer were plunged into steam produced at the normal atmospheric pressure the air would expand, showing an increase of volume equal to $\frac{100}{273}$ of its volume at 0°C . Hence, at 100°C . the air would occupy 373 cm. of the length of the

tube, and if this space of 100 cm., between the freezing point and boiling point, be divided into 100 equal parts of 1 cm. each, then each division will correspond to 1°C . If now this method of division be carried down the tube below the freezing point, the reading at the bottom will be -273°C . This temperature is called the *absolute zero*. If temperatures be reckoned from this point *as zero*, then the freezing point, which is 273°C . higher, is denoted by 273°A. T. ,* and the boiling point is at 373°A. T. , or, generally, $t^{\circ}\text{C}$. becomes $(273 + t)$ on the absolute scale. Since this absolute zero is at the *bottom* of the tube it would seem to imply that, at this temperature, the volume of the air is reduced to nothing. This result is, of course, physically impossible, and only presents itself if we falsely assume that air remains a gas during the whole range of temperature between 0°C . and -273°C .; this we know is not the case, for at a certain temperature it would liquefy, and at a lower temperature become solid; and consequently its law of contraction would altogether change.

In dealing with questions relating to gases it is exceedingly convenient to measure temperatures from this absolute zero, for if we do so we may state that the volume of a given mass of gas is proportional to its absolute temperature. Thus, in the case of the uniform air thermometer tube mentioned above, if the volume of 1 cm. of its length be denoted by v , then the volume of the air at 273°A. T. is $273 v$, at 373°A. T. it is $373 v$, and at $T^{\circ}\text{A. T.}$ it is given by $T v$, that is—

$$\frac{r_t}{r_{t'}} = \frac{273 + t}{273 + t'} = \frac{T}{T'}$$

where T and T' denote absolute temperatures.

This can be derived from relation (ii), Art. 32—

$$\frac{r}{r'} = \frac{1 + c_r t}{1 + c_r t'}$$

Here, $c_r = \frac{1}{273}$, therefore—

$$\frac{r_t}{r_{t'}} = \frac{1 + \frac{1}{273}t}{1 + \frac{1}{273}t'} = \frac{\frac{273 + t}{273}}{\frac{273 + t'}{273}} = \frac{273 + t}{273 + t'} = \frac{T}{T'} \quad (1)$$

* Absolute temperature.

In the same way if we consider the law of variation of pressure with temperature we shall find, that the pressure exerted by a given mass of gas, at constant volume, is proportional to its absolute temperature. Thus, if the pressure at 0°C. be that due to 273 mm. of mercury, then since the pressure at constant volume increases or diminishes by $\frac{1}{273}$ of its value at 0°C. for a rise or fall of 1°C. we have —

$$\begin{array}{lcl} \text{the pressure at } 100^{\circ}\text{C.} & = & 273 + \frac{100}{273} \cdot 273 = 373 \text{ mm.} \\ \text{" " } -1^{\circ}\text{C.} & = & 273 - \frac{1}{273} \cdot 273 = 272 \text{ mm.} \\ \text{" " } -10^{\circ}\text{C.} & = & 273 - \frac{10}{273} \cdot 273 = 263 \text{ mm.} \\ \text{" " } -273^{\circ}\text{C.} & = & 273 - \frac{273}{273} \cdot 273 = 0 \text{ mm.} \end{array}$$

If now we re-write this table and express the temperatures as absolute temperatures we have —

pressure at 273 A. T. is that due to 273 mm. of mercury.

$$\begin{array}{ccccccc} \text{"} & \text{"} & 272 & \text{"} & \text{"} & \text{"} & 272 & \text{"} & \text{"} \\ \text{"} & \text{"} & 263 & \text{"} & \text{"} & \text{"} & 263 & \text{"} & \text{"} \\ \text{"} & \text{"} & 0 & \text{"} & \text{"} & \text{"} & 0 & \text{"} & \text{"} \end{array}$$

This shows that the pressure is directly proportional to the absolute temperature. This result may be deduced directly from relation (iii) of Art. 32 for,

$$\frac{p_t}{p_{t'}} = \frac{1 + c_p t}{1 + c_p t'} = \frac{1 + \frac{1}{273} t}{1 + \frac{1}{273} t'} = \frac{273 + t}{273 + t'} = \frac{T}{T'}. \quad (2)$$

The two relations (1) and (2) which we have thus established are of great importance. It must be remembered that (1) is true only if pressure is constant, and (2) is true only if volume remains constant.

It must also be understood that the absolute zero we have adopted is, in some measure, arbitrary, for we have no experimental knowledge of the actual state of a body at -273°C. and, in reality, the absolute zero chosen is only the lowest reading that could be indicated by an air thermometer, if the properties of air remained unchanged down to that temperature. The complete explanation of the absolute scale of temperature belongs to Thermodynamics, and cannot here be considered; but it is interesting to know that absolute temperature, as determined by the air thermometer, corresponds very closely with absolute temperature

properly so called, as determined from thermo-dynamic considerations.

36. Differential air thermoscopes. Leslie's differential thermoscope, as shown in Fig. 33, consists of a bent tube terminating in two equal bulbs containing air; the bend of the tube contains coloured sulphuric acid, and the quantity of air in the bulbs is so adjusted that, when both are at the same temperature, the columns of acid, on each side of the horizontal part of the bend, are at the same height. Thus constructed the instrument is very sensitive to difference of temperature of the air in the bulbs; the expansion of the air in the warmer bulb depresses the column of acid nearest it and correspondingly raises the other column.

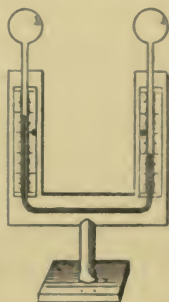


Fig. 33.

Rumford's thermoscope is very similar to Leslie's; the horizontal part of the bend is much longer, and contains a short thread of alcohol, which, acting as an index, takes the place of the sulphuric acid columns.

These instruments are graduated by experiment. The bulbs are placed in water, and the position of the index corresponding to the same temperature in each is marked on the scale. A difference of 10 degrees is then established between the two bulbs and the corresponding index position again marked. The difference of temperature between the two bulbs is now reversed and another mark made on the scale. The space between these two extreme marks is then subdivided as required, and the position of the first mark is taken as the zero of the scale.

CALCULATIONS.

IN connection with the expansion of gases the formulated expressions of the gaseous laws of Art. 32 should be noted :—

(1) **Boyle's Law.** $pv = k$ [Art. 32 (i).]

(2) **Charles' Law.** $v_t = v_o (1 + c_r t)$ [„ „ (ii).]

(3) **Law of pressures.** $p_t = p_o (1 + c_p t)$ [„ „ (iii).]

[In (2) and (3) experiment shows that $c_r = c_p = \frac{1}{273}$ approximately.]

These formulæ are concisely expressed by the relation—

$$\frac{p_1 v_1}{1 + c_r t_1} = \frac{p_2 v_2}{1 + c_r t_2}, \quad [\text{Art. 32 (iv).}]$$

or more conveniently, for the purposes of calculation, by—

$$\frac{p_1 v_1}{T_1} = \frac{p_2 v_2}{T_2}, \quad [\text{Art. 35 (3).}]$$

which is the general expression of formulæ (1) and (2) of Art. 35.

The last formula given involves six quantities, and it is evident that, if any five be given, the remaining one can be calculated. If any one of the variables (p, v, T) involved be supposed constant it cancels out of the equation ; thus, at constant temperature, we have—

$$p_1 v_1 = p_2 v_2 \quad (1 a)$$

at constant pressure,

$$\frac{v_1}{T_1} = \frac{v_2}{T_2} \quad \therefore \frac{v_1}{v_2} = \frac{T_1}{T_2} \quad (1 b). \quad (\text{Compare Art. 35.})$$

and at constant volume,

$$\frac{p_1}{T_1} = \frac{p_2}{T_2} \quad \therefore \frac{p_1}{p_2} = \frac{T_1}{T_2} \quad (1 c). \quad (\text{Compare Art. 35.})$$

When the coefficient of expansion (c_r) is given, formula (iv) of Art. 32 should be adapted in a similar way to the conditions of the problem. (See Ex. 5.)

The approximate relations of preceding chapters should not be used.

EXAMPLES VI.

1. A cubic metre of gas at 760 mm. pressure is subjected, at constant temperature, to a pressure of 2280 mm. Find its volume.

Here, in—

$$p_1 v_1 = p_2 v_2 \quad (1 a).$$

we have—

$$\begin{aligned} p_1 &= 760; p_2 = 2280 \text{ mm.} \\ v_1 &= 1 \text{ cubic metre; } v_2 \text{ is required.} \end{aligned}$$

Hence—

$$760 \times 1 = 2280 v_2.$$

Or—

$$v_2 = \frac{760}{2280} = \frac{1}{3} \text{ cubic metre.}$$

2. A litre of hydrogen, at 10°C. , is heated at constant pressure to 293°C. Find its volume.

Here, in—

$$\frac{v_1}{v_2} = \frac{T_1}{T_2} \quad (1 b).$$

we have—

$$\begin{aligned} v_1 &= 1 \text{ litre; } T_1 = 273 + 10 = 283, \\ T_2 &= 273 + 293 = 566; v_2 \text{ is required.} \end{aligned}$$

Hence—

$$\frac{1}{v_2} = \frac{283}{566} = \frac{1}{2},$$

$$\therefore v_2 = 2 \text{ litres.}$$

3. Air is enclosed in a vessel at 0°C. , and, the volume being kept constant, the temperature is lowered to -88°C. , at which temperature the pressure is found to be 385 mm. Find the pressure at 0°C.

Here, in—

$$\frac{p_1}{p_2} = \frac{T_1}{T_2}$$

we have—

$$p_1 \text{ is required; } T_1 = 273.$$

$$p_2 = 385 \text{ mm.; } T_2 = 273 + (-88) = 273 - 88 = 185.$$

Hence—

$$\frac{p_1}{385} = \frac{273}{185},$$

$$\therefore p_1 = 568.1 \text{ mm.}$$

4. Find the volume occupied at 0°C. and 760 mm. pressure by 500 c.cm. of oxygen measured at 10°C. and 760 mm.

Here, in—

$$\frac{p_1 v_1}{T_1} = \frac{p_2 v_2}{T_2}$$

we have—

$$\begin{aligned} p_1 &= 750 \text{ mm.}; & p_2 &= 760 \text{ mm.} \\ v_1 &= 500 \text{ c.cm.}; & v_2 &\text{ is required.} \\ T_1 &= 273 + 10 = 283; & T_2 &= 273 + 0 = 273. \end{aligned}$$

Hence—

$$\frac{750 \times 500}{283} = \frac{760 v_2}{273}.$$

Or—

$$\begin{aligned} v_2 &= \frac{273 \times 750 \times 500}{283 \times 760} \\ &= 480.635 \text{ c.cm.} \end{aligned}$$

5. A thousand cubic centimetres of air at 50°C. are cooled down to 10°C. , and at the same time the external pressure upon the air is increased from 750 mm. to 765 mm. What is the volume reduced to, the coefficient of expansion of air for 1°C. being 0.00366?

Here, applying formula (iv) of Art. 32, viz.—

$$\frac{p_1 v_1}{1 + c_r t_1} = \frac{p_2 v_2}{1 + c_r t_2},$$

we have—

$$\begin{aligned} p_1 &= 750 \text{ mm.} & p_2 &= 765 \text{ mm.} \\ v_1 &= 1000 \text{ c.cm.} & v_2 &\text{ is required.} \\ t_1 &= 50^\circ \text{C.} & t_2 &= 10^\circ \text{C.} \\ c_r &= 0.00366. \end{aligned}$$

$$\therefore \frac{750 \times 1000}{1 + (0.00366 \times 50)} = \frac{765 v_2}{1 + (0.00366 \times 10)}.$$

$$\therefore \frac{750 \times 1000}{1.183} = \frac{765 v_2}{1.0366}.$$

$$\therefore v_2 = 859.06 \text{ c.cm.}$$

8. In a determination of the coefficient of expansion of air by Gay Lussac's method the volume of the air in the tube was found to be 240 c.cm. at 0°C. , and at 77°C. its apparent volume was 310 c.cm. Find the value obtained for the required coefficient. The mean coefficient of cubical expansion of glass is 0.000026.

11. A porcelain air thermometer is used to determine the temperature of a furnace. The excess of the pressure of the air in the bulb over the atmospheric pressure is found to be that due to 1843 mm. of mercury. Find the temperature of the furnace, given that the barometric height at the time of determination equals 758 mm.

12. A volume V of gas at pressure P and temperature T is heated (1) at constant pressure and (2) at constant volume to a temperature T' . Express in terms of P , V , T , and T' the resulting *state* in each case. [The *state* of a given mass of gas is expressed by giving its pressure, volume, and temperature.]

13. Ten litres of hydrogen at 20°C . and 750 mm. pressure occupy a volume of 9532.4 c.cm. at 10°C . and 760 mm. pressure. Find the mean coefficient of expansion of hydrogen.

17. A litre of hydrogen weighs 0.0896 gram at 0°C . and 760 mm. pressure. Find the weight of a litre at 20°C . and 766 mm. pressure.

18. Compare the density of air at 10°C . and 750 mm. pressure with its density at 15°C . and 760 mm. pressure.

19. A flask containing air is corked up at 20°C . and heated in an air bath. A pressure of two atmospheres inside the flask will force the cork out; at what temperature will this take place?

20. At what temperature will a volume of air at 0°C ., when heated at constant pressure, double its volume?

22. Find the temperature of absolute zero on the Fahrenheit scale.

EXAMINATION QUESTIONS.

QUESTIONS SET AT LONDON UNIVERSITY EXAMINATIONS.

Matriculation.

1. Describe the process of filling and closing a mercurial thermometer. It is sometimes found that when a mercurial thermometer is placed in melting ice, the temperature indicated is not zero, but a fraction of a degree above zero: how is this explained? *Jan., 1876.*

2. Describe an apparatus by which the Coefficient of Expansion of a Gas may be found. 500 cubic centimetres of Oxygen Gas are measured when the temperature is 20°C. , and the temperature is then raised to 40°C. , the pressure meanwhile remaining invariable. What is the volume of the Oxygen at the latter temperature? (The Coefficient of Expansion of Oxygen is $\frac{1}{273}$.) *June, 1876.*

3. A thousand cubic inches of air at a temperature of 30°C. are cooled down to zero, and at the same time the external pressure upon the air is doubled. What is the volume reduced to, the coefficient of expansion of air for 1°C. being 0.00366? *Jan., 1877.*

4. State precisely the law which regulates the connection between the temperature and the pressure and volume of a gas, and show that the statement of this law will be the same, whether we study the increase of volume of a gas whose pressure is constant, or the increase of pressure of a gas whose volume is constant. *June, 1877.*

5. Explain the phrase "mean coefficient of linear expansion of a substance between 0° and t° ;" and show that the coefficient of cubical expansion of a metal is very approximately equal to three times its coefficient of linear expansion. *Ibid.*

6. What is meant by "the point of maximum density" of water? Sketch and describe an apparatus by which this point may be determined. *Jan., 1878.*

7. If a bulb of a thermometer with a fine stem be immersed in hot water, it is sometimes noticed that the mercury falls before it begins to rise. Explain this. *June, 1878.*

8. Describe some accurate method of determining the coefficient of linear expansion of a steel bar.

If the expansion of steel is $\frac{2}{3}$ that of brass under the same change of temperature, what will be the best arrangement of rods of these metals to form a gridiron pendulum? *Ibid.*

9. Why do we generally speak of *degrees* of temperature, but *quantities* of heat?

"If two bodies are in thermal equilibrium with the same body,

they are in thermal equilibrium with one another." How would you prove this statement experimentally? *Jan., 1881.*

10. Define the coefficient of linear expansion of a substance.

Describe an experiment which shows that the coefficient of expansion of some metals is much greater than that of others.

Explain the construction of a (compensating) chronometer balance wheel. *June, 1882.*

11. State the law connecting the volume, pressure, and temperature of a given mass of gas. Describe a method of investigating the relation between the pressure and temperature of a quantity of gas whose volume is kept constant. *Jan., 1883.*

12. Distinguish between the absolute and apparent expansion of mercury contained in a thermometer.

The coefficient of absolute (cubic) expansion of mercury is .00018, the coefficient of linear expansion of glass is .000008. Mercury is placed in a graduated glass tube, and occupies 100 divisions of the tube. Through how many degrees must the temperature be raised to cause the mercury to occupy 101 divisions? *June, 1883.*

13. Why is it necessary to determine the freezing point upon a thermometer before determining the boiling point? In determining the boiling point, why is it necessary to observe the barometer? Is the freezing point affected by the height of the barometer, and if so, why? *Jan., 1884.*

14. Distinguish between the coefficients of apparent and of absolute expansion of mercury, and explain how the latter quantity may be determined. *June, 1884.*

15. How may the relation between the pressure and temperature of a given mass of air at constant volume be determined?

A quantity of air occupies 10 cubic feet at 0° C. and under a pressure of 20 inches of mercury. What will be its volume at 30° C., under a pressure of 1,200 inches of mercury? *Ibid.*

16. A glass flask contains, when full at 0° C., 100 c.cm. of mercury. The coefficient of cubical expansion of glass being 0.000026, and that of mercury 0.00018, find the volume at 100° C. of the mercury driven out when the flask and mercury are heated to 100°. *Jan. 1885.*

17. Explain accurately what is meant by the statement that the coefficient of expansion of air is $\frac{1}{273}$. The volume of a certain quantity of air at 50° C. is 500 cubic inches. Assuming no change of pressure to take place, determine its volume at -50° C. and at 100° C. respectively. *Jan., 1886.*

CHAPTER VI.

CALORIMETRY. ✓

37. WE have already seen that heat must be considered as a physical quantity capable of measurement. Any of the effects of heat may be used as a means of measurement; thus, we might adopt as unit of heat the quantity necessary to convert one gram of ice at 0° C. into water at 0° C., then, *two* units of heat would be the quantity required to convert *two* grams of ice at 0° C. into water at 0° C. This method of measurement is an example of a certain change of state being chosen as a means of measuring quantity of heat. A similar system may be employed, by choosing, as unit of heat, the quantity necessary to convert unit mass of water at 100° C. into steam at 100° C. We might also define the unit of heat as the quantity required to produce a definite increase of volume, or of pressure, in a given mass of some standard substance, in a particular state. Neither of these methods necessarily requires the use of the thermometer.

The method of measurement actually adopted is, however, thermometric, and *the unit of heat*, most commonly chosen, *is the quantity required to raise the temperature of unit mass of water from 0° C. to 1° C.* If the unit mass chosen be the *gram*, then this unit of heat is called the *gram-degree* unit, and similarly for any other unit of mass.

On this system n units of heat is the quantity required to raise n units of mass of water from 0° C. to 1° C.

It should be noticed that we here tacitly assume that, when equal masses of the same substance, in the same state, undergo the same thermal change, then the quantities of heat required to produce this change are equal. This is evidently allowable, but if we had stated, that n units of heat was the quantity required to raise *unit mass* of water

from 0° C. to n° C. then the assumption involved would not be allowable, for, we have no right to assume that the quantity of heat required to raise unit mass of water from 1° C. to 2° C., or from 2° C. to 3° C., is the same as that required to raise it from 0° C. to 1° C., and, in fact, experiment shows that this is not the case though very nearly so.

We shall then adopt as the *unit of heat, the quantity required to raise one gram of water from 0° C. to 1° C.* This unit is sometimes called a *calorie*. For practical purposes it will be necessary to consider the quantities of heat required to raise unit mass of water *one degree* in temperature as equal wherever that degree be chosen on the scale; that is, we must consider the quantity of heat required to raise unit mass of water from t° C. to $(t + 1)^{\circ}$ C. as constant, and equal to the unit chosen above, whatever t may be. Hence, if the temperature of m grams of water is raised from t° C. to t'° C., the gain of heat is $m(t' - t)$ gram-degree units of heat, and if we assume that the water gives out the same amount of heat in cooling from t'° C. to t° C. as is required to raise it from t° C. to t'° C., then, the loss of heat in cooling through this range of temperature is also equal to $m(t' - t)$ gram-degrees.

Let us now consider what takes place when we mix together two quantities of water at different temperatures. To take a concrete example, suppose that 10 grams of water at 10° C. are mixed with 10 grams of water at 50° C.; then (Art. 3) there is a transfer of heat from the water at 50° C. to the water at 10° C., and this transfer goes on until a common temperature is reached. Now, if we suppose that there is no interchange of heat except that between the two quantities of water* we can evidently state, that after the common temperature has been reached

$$\left. \begin{array}{l} \text{The loss of heat by the water} \\ \text{initially at } 50^{\circ} \text{ C.} \end{array} \right\} = \left\{ \begin{array}{l} \text{Gain of heat by the water} \\ \text{initially at } 10^{\circ} \text{ C.,} \end{array} \right.$$

and this relation enables us to determine the common

* This cannot be the case in an actual experiment, for there must be an interchange of heat between the water and the containing vessel.

temperature: for, if we denote it by t , then the 10 grams of warm water have been cooled down from 50°C. to $t^{\circ}\text{C.}$, and therefore the loss of heat is given by $10(50 - t)$ gram-degree units of heat. Similarly, the gain of heat by the cold water is given by $10(t - 10)$ gram-degree units. Thus we have—

$$\begin{aligned} 10(50 - t) &= 10(t - 10), \\ 500 - 10t &= 10t - 100, \\ 20t &= 600, \\ \therefore t &= 30. \end{aligned}$$

And this result we see must be correct, for 10 grams of water in cooling through 20 degrees (from 50°C. to 30°C.) will lose the same amount of heat as is gained by 10 grams of water in being heated through 20 degrees (from 10°C. to 30°C.).

38. Thermal capacity. We have seen (Art. 5) that the application of heat to any body produces effects which depend on the state of the body and the circumstances of the experiment. In what follows we shall deal only with the effects indicated by change of temperature and change of state as explained in Art. 5. Considering for the present only the former of these effects, it is found by experiment, that if equal quantities of heat be imparted, in any way, to *equal* masses of different *substances*, different temperature changes are produced: for example, if 10 gram-degree units of heat be imparted to 1 gram of water, to 1 gram of iron, and to 1 gram of lead respectively, then the temperature of the water will be raised 10 degrees, that of the iron nearly 88 degrees, and that of the lead nearly 323 degrees. It thus appears, that the effect of heat in raising temperature is very different for different substances, and this is expressed by saying, that the **thermal capacity per unit mass*** differs with the substance.

* It would be an advantage if physicists were generally agreed to refer to the *thermal capacity of unit mass* of a substance as the *thermal capacity* of that substance. The relation between the thermal capacity and specific heat of any substance would then be exactly analogous to that between density and specific gravity in the C. G. S. system.

If a quantity of heat H be employed in raising the temperature of unit mass of a substance, from t° C. to t'° C., then the **mean thermal capacity of unit mass of that substance between t° C. and t'° C. is the mean quantity of heat required to raise unit mass of the substance through 1° C.**, and is expressed by

$$c = \frac{H}{t' - t}.$$

We sometimes speak of the *thermal capacity of a body*, meaning the quantity of heat necessary to raise the temperature of *the body* through one degree Centigrade; if the body be of mass m , and the thermal capacity of unit mass of its material be denoted by c , then its thermal capacity is given by mc . The *thermal capacity of unit volume* is also an important constant; the mass of unit volume of a substance is, by definition, the density of that substance, and if this be denoted by d , then the thermal capacity of unit volume is given by cd .

39. Specific heat. The *mean specific heat* of any substance, for a given range of temperature, may be defined as the **ratio of the mean thermal capacity of unit mass of the substance for that range of temperature, to the mean thermal capacity of unit mass of water between 0° C. and 1° C.**

The mean thermal capacity of unit mass of water between 0° C. and 1° C. has been chosen as our unit of heat. Hence we may define the *mean specific heat* of a substance, for a given range of temperature, as the **measure of the mean thermal capacity of unit mass of the substance for the same range of temperature.** Thus, if the mean thermal capacity of unit mass of a substance is 0.9 *gram-degree units of heat* the mean specific heat of that substance is 0.9 . From this it is evident that the mean specific heat of water between 0° C. and 1° C. is 1 , and that, if s denote the mean specific heat of any substance between t° C. and t'° C., the **measure of the thermal capacity of unit volume of that substance, within the given range of temperature, is expressed by sd .** Similarly, the **measure of the thermal**

capacity of mass m of the substance is given by ms , and the quantity of heat required to raise the temperature of this mass from $t^{\circ}\text{C.}$ to $t'^{\circ}\text{C.}$ is given by $Q = ms(t' - t)$ units of heat.* Q also expresses the quantity of heat given out when a mass m of the substance falls in temperature from $t'^{\circ}\text{C.}$ to $t^{\circ}\text{C.}$ This result is important, and may be stated in words thus:—*The measure of the quantity of heat* gained or lost by a body, during a given change of temperature, is equal to the product of the mass of the body, into the specific heat of the substance of the body, into the change of temperature considered.*

We shall have occasion later on to refer to the term **water equivalent**. The water equivalent of any body is the mass of water whose thermal capacity is equivalent to that of the body. Thus, if w denote the water equivalent of a copper vessel, of mass m and specific heat s , we have—

$$\begin{aligned} w \times 1 &= ms \\ \therefore w &= ms \dagger \end{aligned}$$

that is, *the water equivalent of any body is the product of the mass of the body into the specific heat of its material.*

40. Determination of specific heat by the method of mixture.

(1) **Principle of the method.** In this method a known mass of substance, of which the specific heat is to be determined, is raised to a known temperature, and then immersed in a known mass of water, also at a known temperature. A transfer of heat takes place, and goes on until a common temperature, which is indicated by a thermometer placed in the water, is reached. From these data we are able to express the heat lost by the substance and that gained by the water, in terms involving mass,

* The units of heat in which Q is expressed must be **consistent** with those of m ; e.g., if m is given in **grams** then Q must be expressed in **gram-degrees**.

† It should be noticed that, since s is a mere number, w must be of the same dimensions (denomination) as m ; i.e., if m is given in grams then w is also expressed in grams.

specific heat, and change of temperature, and, on equating these expressions (Art. 37), we obtain a relation from which we can calculate the required specific heat of the substance. To take an example, suppose 100 grams of copper at $100^{\circ}\text{C}.$ are dropped into 200 grams of water at $10^{\circ}\text{C}.$, and that, when the transfer of heat is just over, the common temperature of the copper and water is shown, by a thermometer placed in the latter, to be 14° . Then we have by Art. 39—

Loss of heat by copper = $100 \times s \times (100 - 14)$. (1)
where s denotes the specific heat of copper.

Gain of heat by water = $200 \times 1 \times (14 - 10)$. (2)

Then, equating (1) and (2) (Art. 37), we get—

$$\begin{aligned} 100 \times s \times 86 &= 200 \times 4 \\ 8600 s &= 800 \\ s &= \cdot 093 \text{ nearly.} \end{aligned}$$

This, so far, is perfectly simple, but, in practice, we must evidently take account of the fact that, not only does the water gain heat, but also the vessel which contains it (the calorimeter), the thermometer and the stirrer which is used to stir the water during the transfer of heat in order to secure uniformity of temperature throughout its mass. If we know the mass and specific heat of the material of each of these bodies we can proceed in exactly the same way as above to obtain a relation for the determination of s . Thus, to take a general example, suppose m grams of a given substance to be heated to a temperature $T^{\circ}\text{C}.$, and dropped into M grams of water at temperature at $t^{\circ}\text{C}.$, and let the mass of the calorimeter in grams and the specific heat of its material be denoted by m_1 and s_1 , the mass and specific heat of the stirrer by m_2 and s_2 , and the mass and specific heat of the thermometer by m_3 and s_3 respectively. Then, as before, if θ denote the final temperature of the mixture, we have—

Loss of heat by given substance = $m s (T - \theta)$.

Gain of heat—

- (1) By water = $M \times 1 \times (\theta - t)$.
- (2) By calorimeter = $m_1 \times s_1 \times (\theta - t)$.
- (3) By stirrer = $m_2 \times s_2 \times (\theta - t)$.
- (4) By thermometer = $m_3 \times s_3 \times (\theta - t)$.

Hence, total gain of heat =

$$(M + m_1 s_1 + m_2 s_2 + m_3 s_3) (\theta - t) = M' (\theta - t)$$

where

$$M' = M + m_1 s_1 + m_2 s_2 + m_3 s_3.$$

Hence, equating loss and gain of heat, we have—

$$m s (T - \theta) = M' (\theta - t).$$

Or—

$$s = \frac{M' (\theta - t)}{m (T - \theta)} \quad (1).$$

In the expression $M' = M + m_1 s_1 + m_2 s_2 + m_3 s_3$ we see that $M' = M$ *plus* the water equivalents (Art. 39) of the calorimeter, the stirrer, and the thermometer. These may be determined by calculation from their mass and specific heat, but it is evident that only those portions of the calorimeter and thermometer which are in contact with the water can directly receive heat from it, and hence the result obtained by calculation would be too great. For this reason it is generally necessary to determine, by experiment, the water equivalent of the whole calorimetric apparatus under conditions which resemble as closely as possible those under which the specific heat determination is made.

We have, so far, assumed the specific heat required to be that of the solid, but the method is equally applicable to the determination of the **specific heat of a liquid**.

(a) By enclosing the liquid in a thin vessel, raising it to a temperature T , dropping it into water, and proceeding generally in exactly the same way as for a solid. In calculation, allowance would have to be made for the heat given out by the containing vessel; thus, if m grams of a given liquid (specific heat s) be enclosed in a thin glass vessel of mass m' and specific heat s' , then we have—

Loss of heat—

$$(1) \text{ By liquid} = m s (T - \theta).$$

$$(2) \text{ By containing vessel} = m' s' (T - \theta).$$

If m' and s' be known the relation, obtained in the usual way, between loss and gain of heat, serves to determine s .

(b) By placing a liquid, the specific heat of which is required, in the calorimeter and dropping into it a known mass of a solid of known specific heat, then, on equating loss and gain of heat, we obtain, as before, an equation from which we may calculate the specific heat of the liquid. In fact, an equation obtained in this way from the data of any particular experiment gives us a relation from which we may calculate any one of the quantities involved, provided the others are known.

(2) **Practice of the method.** We shall first describe the apparatus employed, and then consider the method of work in determining the specific heat of a given substance.

(a) *The calorimeter and its accessories.* The calorimeter is usually a cylindrical vessel of thin copper which may be about 10 cms. high and 5 cms. in diameter. It is generally surrounded by a larger copper vessel, of similar shape, in which it is either suspended by silk strings or supported on the points of conical shaped pieces of cork placed on the bottom of the outer vessel. This arrangement is adopted to prevent loss of heat by conduction, and by convection and radiation to the surrounding air; to further prevent loss of heat* both the outer surface of the calorimeter and the inner surface of the containing vessel are highly polished, for, it is found by experiment that the brighter the surface of a body, the less rapidly will that body lose heat (by radiation); hence, in this case, the calorimeter will lose heat slowly, and a portion of the little it does lose is reflected back from the bright inner surface of the containing vessel. The thermometer, which should be delicate and of small range, may be placed loosely in the calorimeter or supported in a suitable stand. The stirrer usually consists of a small disc of thin copper or brass perforated with holes and attached to a piece of wire carrying a small wooden handle.

* When a hot body, *e.g.*, a red-hot ball of iron, is suspended in the air, and allowed to cool, it loses heat in two ways: (1) by the convection of the air, and (2) by radiation (Art. 81). Both these sources of loss are guarded against in the above arrangement (1) by enclosing the calorimeter in an outer vessel and (2) by the polished surfaces.

(b) *The heater.* A convenient form of heater is that shown in Fig. 34. It consists of a cylindrical vessel $A B C D$, through the centre of which passes a thin copper tube $a b c d$. The space between the two is quite closed in, and communicates, through the tube i , with a steam boiler; the steam thus admitted into the vessel circulates round $a b c d$ and finally escapes by the tube at o . The tube $a b c d$ is closed at its upper end by a cork, through which passes a thermometer, t , and a fine silk thread, for the suspension of the body to be heated. The whole apparatus is fixed to the stand $S S'$, and may be turned round this stand into the position shown by the dotted lines in the figure, so as

to be vertically over the calorimeter at C .

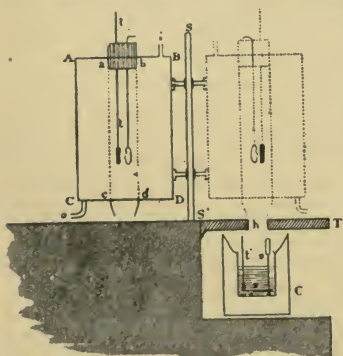


Fig. 34.

To employ this apparatus for the determination of the specific heat of a solid substance, a portion of the substance, having as large a surface as possible, is *loosely* rolled or bent into a small compact mass, weighed and attached to the thread in $a b c d$. The position of the thermometer, t , is such that its final reading can just be seen above the cork. The

length of the silk cord is then adjusted so that the substance is suspended close to the bulb of the thermometer. The steam is now admitted into the heater and allowed to pass until some time after the thermometer indicates a constant temperature, which will be less than that of the steam. In the meantime the calorimeter is got ready for use. It is first weighed, then filled rather more than half full of water, and again weighed; the difference of the weights thus obtained determines the mass, M , of the water. The stirrer, s , and thermometer, t' , are now placed in position, and the whole allowed to remain until

the temperature becomes quite constant. When this is the case, and when the thermometer in the heater has been stationary for some time, both temperatures are noted, giving t and T of (2) above. The calorimeter is then placed as at C in Fig. 34, and the heater swung round into the position shown by the dotted lines so as to be vertically over C. Then, by loosening the string which supports the substance in the heater, the mass falls through the hole, h , in the table or shelf, T , into the calorimeter, which is at once removed and the reading of the thermometer watched while the water is kept well stirred; for a short time the temperature continues to rise, then it becomes stationary, and finally begins to fall. The reading is taken when the temperature is stationary, and thus θ , of equation (1) of preceding article, is determined. If we consider this equation we see that all the data for calculating s have been determined except M' . Now $M' = M + \text{water equivalent of calorimeter, etc.}$, and M has been determined, therefore it only remains to determine the water equivalent of the calorimeter and its accessories. This, as we have explained, must be done by experiment, and the conditions of the experiment must be, as nearly as possible, those of the specific heat determination. For this purpose a known mass of water, very nearly equal to M , is placed in the calorimeter and allowed to take a temperature as near t as possible. A quantity of hot water is then slowly added until the temperature rises nearly to θ ; the whole is then well stirred, and the final temperature, which will be somewhere near θ , noted. From the determination of the quantity of water first put in the calorimeter, the weight of the calorimeter and the weight of the water it contained will be known, and if it is again weighed, at the end of the experiment, the difference in weight will determine the mass of hot water which has been added. Let this mass be denoted by m_1 , and its temperature by T_1 ; also let M_1 denote the mass of water first placed in the calorimeter, t_1 its temperature, and θ_1 the final temperature of the mixture. Then, proceeding in the usual way,

$$\text{Heat lost by hot water} = m_1(T_1 - \theta_1).$$

Heat gained by—

$$(a) \text{ Water in calorimeter} = M_1(\theta_1 - t_1),$$

$$(b) \text{ Calorimetric apparatus} = w(\theta_1 - t_1),$$

where w represents the water equivalent of the calorimeter and its accessories, that is, it represents the mass of a quantity of water, which would absorb a quantity of heat equal to that absorbed by the calorimeter, etc., and which may therefore be added to M to give M' . [Art. 40 (1).]

Hence, equating we have—

$$m_1(T_1 - \theta_1) = M_1(\theta_1 - t_1) + w(\theta_1 - t_1);$$

or—

$$w(\theta_1 - t_1) = m_1(T_1 - \theta_1) - M_1(\theta_1 - t_1),$$

$$w = m_1 \frac{(T_1 - \theta_1)}{(\theta_1 - t_1)} - M_1.$$

In adding the hot water to the water in the calorimeter, special precautions have to be taken to prevent cooling. It is generally run out, through a fine copper tube, from the inner compartment of a small double vessel somewhat similar in construction to the heater described above.

The water equivalent thus determined for any calorimetric apparatus should be noted, and whenever the apparatus is used it will only be necessary to add w to the mass of water placed in the calorimeter to get the mass of water (M') which is equal in thermal capacity to that of the water and the apparatus.

There is one other precaution which it is necessary to take, in these experiments, to prevent loss of heat by cooling. Matters should be so arranged that the temperature of the room may lie midway between t and θ . Thus, if the temperature of the room is 12°C. and we expect a rise of temperature, from $t^\circ\text{C.}$ to $\theta^\circ\text{C.}$, of about 4 degrees, then t° should be adjusted to 10°C. In this way the calorimeter gains heat from the air of the room in rising from 10°C. to 12°C. , and gives out a nearly equal quantity in rising from 12°C. to 14°C. This arrangement can only be effected in a second or third determination when

a previous experiment has served to approximately determine θ .

If the substance, the specific heat of which is required, is a solid in small pieces they should be suspended in the heater in a basket made of fine brass wire; or, if a liquid, it may be enclosed in a thin glass vessel or placed in the calorimeter, as explained above. In all cases, the method of experiment is similar to that described above, and the principle of calculation is the same—viz., heat lost on one side is equated to heat gained on the other.

42. Specific heat of gases. Regnault made careful determinations of the specific heat of various gases. The gas was collected into a reservoir, and then heated to a high temperature by passing it through a long spiral tube immersed in an oil bath. It then passed through a calorimeter where it was cooled to the temperature of the surrounding water, and then escaped into the air. The rise of temperature in the calorimeter was noted, and the mass of gas which had passed through was determined by the change of pressure in the reservoir. On equating the loss of heat of the gas to the gain of heat of the calorimeter and the contained water, a relation was obtained which served to determine the specific heat of the gas. Regnault's experiments established the following results:—

1. The specific heat of a gas at constant pressure is the same whatever that constant pressure may be.
2. All simple gases have, at the same pressure, the same thermal capacity per unit volume.
3. The specific heat of a gas is the same at all temperatures.

It should be noticed that gases have two specific heats, according as they are heated at constant pressure and allowed to expand (specific heat at constant pressure) or at constant volume without expansion (specific heat at constant volume). The specific heat at constant pressure is greater than the specific heat at constant volume, because, not only has the temperature of the gas to be raised,

but a quantity of heat, equivalent to the work done during the expansion against the external pressure, has to be supplied to the gas.

TABLE OF SPECIFIC HEATS.

Solids (mean specific heat between 10° C. and 100° C.).

Copper	·0951	Antimony	·0508
Silver	·0570	Brass	·0939
Iron	·1138	Magnesium	·2499
Zinc	·0955	Aluminium	·2143
Tin	·0562	Glass	·1877
Lead	·0314	Ice	·5040
Gold	·0324	Sulphur	·1777
Platinum	·0324	Graphite	·2008
Bismuth	·0308	Diamond	·1469

Liquids.

Mercury	·0333	Alcohol	·615
Sulphuric acid	·343	Oil of turpentine	·462
Ether	·503	Acetic acid	·659

Gases (at constant pressure).

Air	·2374	Nitrogen	·2438
Hydrogen	3·4090	Carbonic anhydride	·2163
Oxygen	·2175	Carbonic oxide	·2450
Chlorine	·1210	Steam	·4805

43. Properties of a typical thermometric substance.

We are now in a position to consider this question more fully than in Chap. II. Assuming that change of temperature is indicated by change of volume of a given substance, the chief properties necessary to constitute the substance a typical one are:—

1. *Uniform expansion with increase of temperature* (Art. 26).

2. *A large coefficient of expansion*; that is, the increase of volume for 1° rise of temperature should be sufficiently large to be capable of accurate measurement.

3. *Low thermal capacity per unit volume*. When this is the case—together with (2)—the thermometer is very sensible to loss or gain of heat.

4. *Constant specific heat at all temperatures.* This is a very important requirement. A substance possessing this property would lose or gain equal quantities of heat for each degree change of temperature. For most substances the specific heat increases with the temperature.

5. *The range of temperature, within which the above properties remain unchanged, should be as wide as possible.*

6. *Its power of transmission of heat, either by conduction or convection (Art. 93) should be great.* This insures rapid and uniform distribution of temperature throughout its substance.

The only substance possessing the above properties in the required degree is air, or one of the gases which closely follow Boyle's law, and, for this reason, the air thermometer is the most perfect instrument of its kind. The properties of mercury, between 0°C. and 100°C. , very closely conform to those given above.

CALCULATIONS.

44. THE following results, obtained in the preceding chapter, should be noticed :—

1. $Q = m s (t' - t)$ (Art. 39). Q denotes the quantity of heat lost or gained by a body of mass m and specific heat s during change of temperature from t to t' .

2. $w = m s$ (Art. 39). w denotes the water equivalent of a body of mass m and specific heat s .

3. Principle of calculation for problems on method of mixture (Art. 40). The method of mixture always involves the loss of heat by one portion of the system considered, and a corresponding gain of heat by the remaining portion. If the former portion be represented by A, and the latter by B, the principle of calculation for the method may be stated thus—

$$\left. \begin{array}{l} \text{The loss of heat by A during} \\ \text{the change from its initial} \\ \text{to its final state} \end{array} \right\} = \left\{ \begin{array}{l} \text{The gain of heat by B during} \\ \text{the change from its initial} \\ \text{to its final state.} \end{array} \right.$$

In applying this the following points must be attended to: (a) There must be no loss or gain of heat from without. (b) The total mass of the system must be constant throughout. This condition is really involved in (a).

EXAMPLES VII.

1. Ten grams of water at 98° C. are poured into a copper vessel, weighing 25 grams and containing 100 grams of water at 6° C. Find the final temperature of the mixture. Specific heat of copper = 0.092.

Here, if θ denote the final temperature, we have—

The heat lost by the ten grams of water initially at 98° C.
 $= 10(98 - \theta)$ units.

Heat gained.—(1) By copper vessel = $25 \times 0.092(\theta - 6)$ units
 $= 2.3(\theta - 6)$ units.

(2) By water in copper vessel = $100(\theta - 6)$.

Hence, equating we get

$$10(98 - \theta) = 102.3(\theta - 6).$$

$$\therefore 112.3 \theta = 1593.8, \text{ or } \theta = 14.2^{\circ} \text{ C. (nearly).}$$

2. In order to determine the specific heat of silver, a piece of the metal, weighing 21 grams, is heated to 98°C . and then dropped into a calorimeter containing 100 grams of water at 10°C . The final temperature of the mixture is 11°C .; find the specific heat of silver. The water equivalent of the calorimetric apparatus is 3.6 grams.

Here, if s denote the specific heat of silver, we get—

Heat lost by the silver $= 21 \times s \times (98 - 11) = 1827s$ units.

Heat gained—

(a) By calorimetric apparatus $= 3.6(11 - 10) = 3.6$ units.

(b) By water in calorimeter $= 100(11 - 10) = 100$ units.

Hence, equating we get—

$$1827s = 103.6$$

$$\therefore s = \frac{103.6}{1827} = 0.0567.$$

3. Ten grams of common salt, at 91°C ., having been immersed in 125 grams of oil of turpentine* at 13°C ., the temperature of the mixture was 16°C . Find, from these data, the specific heat of common salt, supposing no loss or gain of heat to have taken place from without and taking the specific heat of oil of turpentine as 0.428.

Here we have—

$$10 \times s \times (91 - 16) = 125 \times 0.428 \times (16 - 13)$$

$$\therefore 750s = 160.5$$

$$\therefore s = \frac{160.5}{750} = 0.214.$$

4. A mass of 200 grams of copper, whose specific heat is 0.095, is heated to 100°C ., and placed in 100 grams of alcohol at 8°C ., contained in a copper calorimeter, whose mass is 25 grams, and the temperature rises to 28.5°C . Find the specific heat of the alcohol.

The heat lost by the copper

$$= 200 \times 0.095 \times (100 - 28.5)$$

$$= (200 \times 0.095 \times 71.5) \text{ gram-degrees.}$$

The heat gained—

$$(a) \text{ By the calorimeter } = 25 \times 0.095 \times (28.5 - 8)$$

$$= (25 \times 0.095 \times 20.5) \text{ gram-degrees.}$$

$$(b) \text{ By the alcohol}$$

$$= (100 \times s \times 20.5) \text{ gram-degrees,}$$

where s denotes the specific heat of the alcohol. Then, equating total loss and gain of heat, we have—

$$(200 \times 0.095 \times 71.5) = (25 \times 0.095 \times 20.5) + (100 \times 20.5 \times s),$$

$$1358.5 = 48.6875 + 2050s.$$

$$\therefore 2050s = 1309.8125.$$

$$\therefore s = 0.6389.$$

* Water could not be used in this case as common salt is soluble in water. This point should be noticed.

5. The following data were obtained in an experiment for the determination of the water equivalent of a given calorimetric apparatus.

Weight of apparatus	45.623 grams.
" " " + water	224.583 "
Initial temperature of apparatus and water	9° C.
Temperature of hot water	78° C.
Final temperature	13.2° C.
Weight of apparatus after addition of hot water	236.493 "

[*Note.*—These data are given in the order of their determination in an actual experiment.]

Here—

Weight of water in calorimeter = $224.583 - 45.623 = 178.960$ grams,
and weight of hot water added = $236.493 - 224.583 = 11.91$ grams.

Therefore, if w denote the water equivalent of the apparatus, we have—

$$\begin{aligned}(178.96 + w)(13.2 - 9) &= 11.91(78 - 13.2), \\ 4.2(178.96 + w) &= 11.91 \times 64.8, \\ 751.632 + 4.2w &= 771.768, \\ \therefore 4.2w &= 20.136. \\ \therefore w &= 4.794 \text{ grams.}\end{aligned}$$

6. A ball of platinum, whose mass is 200 grams, is removed from a furnace and immersed in 150 grams of water at 0° C. If we suppose the water to gain all the heat which the platinum loses, and if the temperature of the water rises to 30° C., what is the temperature of the furnace? Specific heat of platinum is 0.031.

Here, if T denote the temperature of the furnace, we have—

$$\begin{aligned}200 \times 0.031(T - 30) &= 150 \times 30. \\ \therefore 6.2(T - 30) &= 4500. \\ \therefore 6.2T &= 4500 + 186. \\ \therefore 6.2T &= 4686. \\ \therefore T &= \frac{4686}{6.2} = 755.8^\circ \text{C.}\end{aligned}$$

[This example indicates a method of measuring very high temperatures.]

9. 280 grams of zinc (specific heat = .095) are raised to the temperature 97° and immersed in 150 grams of water at 14° contained in a copper calorimeter weighing 96 grams. The specific heat of copper being .095, what will be the temperature of the mixture supposing that there is no exchange of heat except among the substances mentioned? What is the water equivalent of the calorimeter employed?

10. A copper vessel containing a thermometer is at 12° C.; 50 grams of water at 60° are poured in, and the temperature, after stirring, is found to be 50°: find the thermal capacity, or water equivalent, of the vessel and thermometer.

12. Determine the specific heat of copper from the following data :

Weight of copper	16.65 grams.
" " water in calorimeter	49 "
Initial temperature of copper	99.5° C. "
" " " water and calorimeter.	12° C.
Final " " mixture	14.5° C.
Water equivalent of calorimeter, etc.	2.1 grams.

13. Determine the specific heat of alcohol from the following data :

Weight of copper calorimeter	20.4 grams.
" " " + alcohol	70.5 "
" " " dropped into calorimeter	10.5 "
Initial temperature of calorimeter and alcohol	10° C.
" " " copper	98° C.
Final " " " mixture.	12.6° C.

15. A piece of platinum, weighing 120 grams, is taken from a furnace and at once dropped into 100 grams of water at 10° C., contained in a copper vessel, weighing 21 grams. The final temperature is found to be 37° C.: find the temperature of the furnace.

16. 100 grams of mercury at 250° C. are mixed with 80 grams of mercury at 15° C. in a glass vessel weighing 35 grams. Find the final temperature of the mixture.

17. Regnault found that 100.5 units of heat were required to raise the temperature of unit mass of water from 0° C. to 100° C., and 203.2 units to raise its temperature to 200° C. Find the mean specific heat of water between 0° C. and 100° C., between 100° C. and 200° C., and between 0° C. and 200° C.

CHAPTER VII.

CHANGE OF STATE.

LIQUEFACTION AND SOLIDIFICATION.

45. IN general, when a solid body is continuously heated, the temperature rises regularly for some time until a particular temperature is reached. At this temperature the solid begins to melt, and during the process of melting or liquefaction the temperature remains perfectly constant. When the last particle of the solid has become liquid the temperature again begins to rise, and continues to do so until another higher temperature is reached, when it again becomes stationary, and remains so until all the liquid is converted into vapour. So long as this process continues the temperature remains constant, but when complete, the temperature of the vapour thus formed will rise with the continued application of heat.

These phenomena, resulting from the continued application of heat to a substance originally solid, are produced in the inverse order, if, while the other conditions of the experiment remain unchanged, the thermal conditions are reversed. Thus, if the vapour referred to above, after having been heated some degrees higher than the temperature at which vaporization took place, be slowly cooled, its temperature will fall until *condensation* sets in at the temperature at which vaporization was effected, and during condensation the temperature remains constant. When all the vapour has been converted into liquid, that is, when condensation is complete, the temperature again begins to fall, and continues to do so until the temperature of liquefaction is reached. At this temperature, *solidification* commences, and continues, at constant temperature, until all the liquid is converted into solid. After this the

temperature continues to fall so long as cooling goes on. These changes are supposed to take place under the influence of heat in all substances, but the temperatures of liquefaction and vaporization for different substances are widely different, and in some substances our means of producing heat and cold are too limited to effect the changes. Thus, iron melts at a very high temperature, platinum at a still higher, and although carbon has been somewhat softened it has not been thoroughly liquefied. On the other hand, many of the gases condense only at very low temperatures and solidify at temperatures much lower, and it is only in recent years that the so-called permanent gases, oxygen, hydrogen, nitrogen, air, etc., have been liquefied, and, in some cases, solidified. Although our range of observation is thus somewhat limited by our means of producing heat and cold, yet we have sufficient, and continually increasing evidence, that every substance exists in three states—the solid, liquid, and gaseous, though the range of temperature corresponding to any particular state is very different for different substances. When we speak of a substance as a solid, liquid, or gas we refer to its condition at ordinary temperatures.

We have seen that *during* change of state the temperature remains constant, although the substance is either gaining or losing heat. This implies that during one phase of the change the substance absorbs a quantity of heat without experiencing any rise of temperature, and, during the inverse phase of the change, it gives out a quantity of heat without indicating any corresponding fall of temperature. Heat thus absorbed or emitted, at constant temperature, during change of state has been called **latent heat**.

46. Liquefaction and solidification. Experimental investigation shows that the change from the solid to the liquid state is subject to a few simple laws which are also applicable to the inverse change from the liquid to the solid state.

These are given below, in terms applicable to the nature of the change.

I. Laws of liquefaction or fusion.

(i) Each substance begins to melt at a definite temperature which is constant for the same substance under the same conditions. This temperature is called the **melting point**, and is one of the specific constants of that substance, serving to distinguish it from other substances.

(ii) The rate at which liquefaction takes place is proportional to the supply of heat, but the temperature remains constant from the time when fusion commences until it is complete.

(iii) If a substance contract on fusion, *e.g.* ice, its melting point is lowered by pressure, but if it expand, *e.g.* beeswax, its melting point is raised by pressure.

(iv) Each substance during fusion absorbs, per unit mass, a definite quantity of heat which is constant for the same substance under the same conditions. The heat thus absorbed by any substance is called its **latent heat of fusion**.

II. Laws of solidification.

(i) Each substance begins to solidify at a definite temperature which is constant for the same substance under the same conditions, and is the same as the melting point. [This temperature, in the case of water, is called the freezing point—no general name has been given to it.]

(ii) The rate at which solidification takes place depends on the rate at which the substance loses heat, but the temperature remains constant from the time when solidification commences until it is complete.

(iii) If a substance expand on solidification, *e.g.* water, the temperature of solidification is lowered by pressure, but if it contract, *e.g.* beeswax, its temperature of solidification is raised by pressure.

(iv) Each substance during solidification gives out, per unit mass, a definite quantity of heat which is constant for the same substance under the same conditions. [This is also called the latent heat of *fusion*, because the heat is supposed to become latent *during fusion*, and is restored on solidification.]

These laws of fusion and solidification correspond exactly to one another and need only be learnt in one form ; they

have been given here in full, that the student may understand as clearly as possible the nature of the changes to which they refer.

There are some substances which do not exactly conform to these laws; of these, glass is a well-known example—in passing from the solid to the liquid state it passes through an intermediate *viscous* state where it gradually softens, and becoming less and less viscous, finally becomes liquid at a temperature much higher than that at which it began to soften. There are also some instances of solids passing directly into the gaseous condition (*e.g.* iodine) and of gases or vapours passing directly into the solid state (*e.g.* sulphur vapour). This process is called *sublimation*.

47. Determination of the melting point of a substance.

The method of determining the melting point of a given substance depends on the nature of the substance—for example, the determination of the melting points of mercury and glass would require somewhat different methods, both because the temperatures are widely different, and because the nature of the change of state is different in the two cases (Art. 46). We shall describe a method which is applicable where the melting point lies between 0°C. and 100°C. A small quantity of the substance whose melting point is to be determined, is put into a fine capillary tube about 5 cm. long, and closed at one end. This tube is then attached, by small india-rubber bands, to the bulb and lower portion of the stem of a thermometer. This arrangement is then fixed in a suitable stand and placed in a beaker of water in such a position that the open end of the capillary tube is just above the surface of the water. The water in the beaker is slowly heated, and the substance carefully watched. At a certain stage of the experiment melting will suddenly take place; the temperature of the thermometer should then be read, and noted as the *approximate* melting point of the substance. The experiment should now be repeated, and as the temperature approaches the melting point previously determined the heating should

be very gradual. The temperature at which melting is now noticed will be, very approximately, the melting point of the substance.

By using oil in a suitable bath this method of observation may be extended to temperatures much higher than 100°C .

TABLE OF MELTING POINTS.

Platinum	2000°C .	Glass	1100°C .
Wrought iron . . .	1550°C .	Borax	1000°C .
Gold	1200°C .	Cane sugar . . .	160°C .
Cast iron	1200°C .	Sulphur	111°C .
Copper	1090°C .	White wax . . .	68°C .
Silver	1000°C .	Paraffin	54°C .
Zinc	360°C .	Spermaceti . . .	44°C .
Lead	330°C .	Phosphorus . . .	43°C .
Cadmium	320°C .	Butter	33°C .
Bismuth	267°C .	Lard	33°C .
Tin	230°C .	Ice	0°C .
Sodium	90°C .	Bromine	-21°C .
Potassium	58°C .	Mercury	-40°C .

49. Latent heat of fusion. Latent heat has already been explained in general terms, and the latent heat of fusion is defined in Law (iv) above. We may here define it for any substance as *the quantity of heat absorbed by unit mass of that substance in passing from the solid to the liquid state without change of temperature.* The heat thus absorbed is chiefly spent in doing molecular work in the substance. As we have seen (Art. 2) heat is considered to be a form of energy, and, as such, is capable of doing work; when heat is applied to a body, and causes a change of temperature, it is assumed that the kinetic energy of the molecules of the body is increased; but when heat produces a change of state, internal molecular work is done in establishing the new state of aggregation, and external work is done in effecting the accompanying change of volume against the external atmospheric pressure. The so-called latent heat has thus its equivalent in the increased molecular potential energy of the substance in its new state, and for this reason a better name for latent heat would be potential heat.

50. Determination of the latent heat of water. The number of units of heat required to convert 1 gram of ice at 0°C. into water at 0°C. is called the latent heat of water. If L denote the latent heat of water, then the heat absorbed during the conversion of m grams of ice at 0°C. into water at 0°C. is denoted by mL . If the gram be taken as unit of mass, then L is very approximately equal to 80 gram-degree units of heat.* In order to determine the latent heat of water a calorimetric method, very similar to the method of mixture, is adopted. The calorimeter is prepared in the usual way, by placing in it a known quantity of water at a known temperature. A number of small well-dried pieces of ice are then added, and stirred round until completely melted. When this is the case the thermometer is read, and the calorimeter again weighed, in order to determine the weight of ice added. From these data, knowing the water-equivalent of the calorimeter, we can calculate the latent heat of water. Thus let M denote the mass of water in the calorimeter, t its temperature, w the water-equivalent of the calorimeter, θ the final temperature of the water after the melting of the ice, and m the mass of ice added. Then—

(a) Heat lost by water and calorimeter $= (M + w) (t - \theta)$.

(b) Heat absorbed by ice during change of state $\left\{ \begin{array}{l} \text{from ice at } 0^{\circ}\text{C. to water at } 0^{\circ}\text{C.} \end{array} \right\} = mL$.

(c) Heat gained by water thus produced in being $\left\{ \begin{array}{l} \text{raised from } 0^{\circ}\text{C. to } \theta^{\circ}\text{C.} \end{array} \right\} = m\theta$.

But, equating $a = b + c$, we have—

$$(M + w) (t - \theta) = mL + m\theta.$$

Or—

$$mL = (M + w) (t - \theta) - m\theta.$$

Or—

$$L = \frac{(M + w) (t - \theta)}{m} - \theta.$$

* In expressing quantity of heat in particular units, care must be taken to choose a unit consistent with the unit of mass employed in the same connection. For example, if we define the latent heat of water at 0°C. , as the quantity of heat required to convert 1 gram of ice at 0°C. into water at the same temperature then L must be expressed in gram-degrees.

TABLE OF LATENT HEATS OF FUSION.

Ice	80.0	Tin	14.2
Nitrate of sodium	63.0	Cadmium	13.6
Nitrate of potassium (nitre)	47.4	Bismuth	12.6
Zinc	28.1	Sulphur	9.4
Silver	21.1	Lead	5.4
Bromine	16.2	Phosphorus	5.0
Iodine	11.7	Mercury	2.8

51. Solution. When a solid is dissolved in a liquid we have, in general, two distinct effects produced :

- (a) The physical change of state of the solid ;
- (b) Chemical combination.

In solution, properly so called, only (a) takes place, and the process is therefore always accompanied by a fall of temperature due to the absorption of the latent heat of fusion from the solution. For example, the solution of sodium sulphate or ammonium nitrate in water lowers the temperature of the solution to about $15^{\circ}\text{C}.$, and for this reason these solutions are often used as freezing mixtures. In the majority of cases, however, chemical action takes place as well as simple solution, and we then have two thermal effects corresponding to (a) and (b) above. The change referred to in (a) is always accompanied by absorption of heat, and that of (b) always *produces* heat, and thus, the final effect depends on whether the absorption of heat during (a) is greater, equal to, or less than the evolution of heat during (b). For example, a mixture of 4 parts of sulphuric acid and 1 part of snow produces a *rise* in temperature of $90^{\circ}\text{C}.$, but a mixture of 1 part of acid and 4 parts of snow produces a fall of temperature of $-20^{\circ}\text{C}.$

A **freezing mixture** is one in which a low temperature is produced by the absorption of heat resulting from the change of state of some constituent of the mixture. It must be understood that the low temperature of the mixture is only temporary ; it is produced *during* the change of state in the mixture and is complete when that is complete.

The simplest freezing mixtures are those referred to above, where the fall of temperature results from simple solution. A common mixture for laboratory purposes is salt and ice,

or salt and snow. In this mixture both constituents change state, and the temperature resulting from this double absorption of heat is -22°C ., which is the freezing-point of a definite compound of salt and water.

A mixture of 4 parts of chloride of calcium with 3 of snow produces the low temperature of -51°C .

52. Ice calorimeters. We have seen that when 1 gram of ice is melted 80 gram-degree units of heat are absorbed. This fact has been applied in the construction of ice calorimeters for the determination of specific heat. With the exception of that of Bunsen (described below) they are not now much in use. We shall describe two forms.

1. *The ice calorimeter of Laplace and Lavoisier.* This calorimeter (Fig. 35) consists of three vessels. The inner one, A, is made of thin copper, and is used only as a receptacle for the heated body. The middle vessel, B, inside which A is placed, is the calorimeter proper; the space between A and B is filled up with small pieces of ice, which are melted by the heat lost by the heated body in A in cooling down to zero. The vessel B is surrounded by an outer larger vessel, C, and the space between B and C is filled up with ice; this arrangement prevents the melting of the ice in B by heat absorbed from the surrounding air. A tube, t , passes from the bottom of B through C down to a small vessel, D, placed to receive the water resulting from the melting of the ice in B. The tube t' is used merely to run off the water which collects in the ice-jacket C. To determine the specific heat of a substance by means of this calorimeter, a known mass, m , of the substance is heated to $T^{\circ}\text{C}$. in a heater (Art. 40), and dropped into A. The ice in B is melted, and the water produced runs out into the

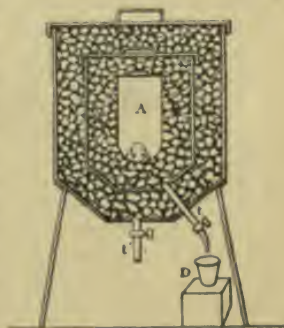


Fig. 35.

vessel D. The mass of water thus produced is determined by weighing, and from these data the specific heat of the substance can be calculated. Thus, if m' denote the mass of water produced, and L the latent heat of water we have, since the final temperature of the substance is zero—

$$ms(T - 0) = m' L, \text{ or } msT = m' L.$$

$$\therefore s = \frac{m' L}{m T}.$$

This calorimeter is simple in principle, but it does not give accurate results, because the mass of the water which runs out into D is not equal to the water produced in B.

2. *Bunsen's ice calorimeter.* The action of this calorimeter depends upon the fact that ice contracts on melting. One

gram of ice has a volume of 1.0908 c.cm., and the volume of the same mass of water at 0°C . is 1.00012 c.cm.* Hence, when 1 gram of ice is melted, and becomes 1 gram of water at 0°C ., a diminution of volume equal to 0.09068 c.cm. is produced, and indicates the absorption of 80 gram-degree units of heat. The calorimeter (Fig. 36) consists of a glass vessel A, having a test tube B sealed into it, as shown in the figure. At its lower end, A passes into the tube tt' , which is bent up, as shown in figure, and, at t' , is fitted with a cork carrying a bent piece of capillary tubing shown at c .

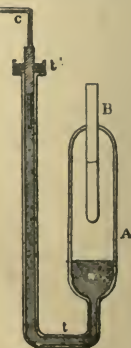


Fig. 36.

This horizontal part of this tube is carefully calibrated, and graduated so that the volume between any two divisions is accurately known.

To prepare this apparatus for use, the upper part of A must be filled with water, and the lower part and the communicating tube with mercury. The instrument is then placed in a suitable vessel, surrounded, as completely as possible, with snow or melting ice and a current of alcohol,

* 1 gram of water at 4°C . has a volume of 1 c.cm., for a gram is, by definition, the weight of 1 c.cm. of water at 4°C . As water expands from 4°C . to 0°C . the volume of a given mass will be slightly greater at the latter temperature.

which has been cooled by a freezing mixture of salt and ice, is passed through the test tube B. The result of this is that the water, in A, surrounding the test tube gets cooled, and finally forms into a solid mass of ice. The test tube B is now cleaned out, and a small quantity of water placed in it; this water soon takes the temperature 0°C. , and the position of the end of the mercury column in the horizontal branch of the tube *c* becomes stationary. Its position is then read.

A known mass of the substance, the specific heat of which is required, is raised to a known temperature in a suitable heater, and then dropped into the water in B. The heat given out by this substance melts a quantity of ice in A, and a consequent diminution of volume is produced, and is indicated by the motion of the end of the mercury column in *c* towards the calorimeter. When the position of this column becomes stationary a second reading is made, and the difference of the two readings thus taken on *c*, determines the decrease in volume caused by the melting of ice in A. Let the decrease of volume thus determined be denoted, in cubic centimetres, by v . Then, since the melting of 1 gram of ice in A would produce a diminution of volume equal to

0.09068 c.cm. , the mass of ice melted in A must equal $\frac{v}{0.09068}$

grams, and as each gram of ice requires 80 gram-degree units of heat to melt it, the quantity of heat given out by

the substance must equal $\frac{80v}{0.09068}$ or $882v$ gram-degree

units of heat. But if m denote the mass of this substance in grams, T the temperature to which it was raised and s its specific heat, then the heat it gives out in cooling to 0°C. is equal to msT gram-degree units. Thus, we have—

$$882v = msT.$$

$$\therefore s = \frac{882v}{mT}.$$

This instrument, when once put in working order, may be used to make several determinations and gives accurate results, but it is rather troublesome to fill and prepare for use.

It is evident that if we work with a substance of known specific heat we may use this apparatus to determine the latent heat of water, or to show that ice contracts on melting, and, by measuring the contraction, to determine the specific gravity of ice. Bunsen made both these determinations, and found 80.025 as the latent heat of water, and 0.91674 as the specific gravity of ice.

CALCULATIONS.

53. THE following points referred to in the preceding chapter should be noticed.

1. If L denote the latent heat of fusion of a given substance, then the quantity of heat **absorbed during fusion** by a mass m of that substance is represented by mL , and the quantity of heat **evolved during solidification** of a mass m of the substance is also represented by mL .

[In each of these cases the temperature of the substance remains constant during the change of state, but the heat absorbed, or given out, affects the temperature of adjacent substances.]

2. **Bunsen's ice calorimeter.** The formula $s = \frac{882v}{mT}$ should *not* be used for the solution of problems. It is however convenient to know it for the verification of answers obtained. It is also helpful to remember that a decrease of volume, v , in the calorimeter indicates, approximately, the melting of 11v grams of ice and a consequent absorption of 11v L gram-degree units of heat, where L denotes the latent heat of water. Problems generally contain all necessary data, and are best worked out from the given data by the method explained in Art. 52.

EXAMPLES VIII.

1. Ten grams of ice at -10° C. are mixed with 120 grams of water at 80° C. Find the final temperature of the mixture. (Specific heat of ice = 0.5 and latent heat of water = 80.)

Here, if θ denote the final temperature we have—

Loss of heat by water = $120(80 - \theta)$.

Gain of heat by—

(a) Ice during change of temperature from -10° C. to 0° C.
= $(10 \times .5 \times 10)$ units = 50 units.

(b) Ice during change of state from ice at 0° C. to water at 0° C.
= 10×80 units = 800 units.

(c) Water thus produced during change of temperature from 0° C. to θ° C. = 10θ .

Hence, equating we get—

$$120(80 - \theta) = 50 + 800 + 10\theta.$$

$$\therefore 130\theta = 8750, \text{ or } \theta = 67.3^{\circ}.$$

2. How many units of heat would cause a mixture of ice and water to contract by 50 c.mm., if 100 c.mm. of water at 0° C. become 100 c.mm. on freezing?

Here, the contraction resulting from the *production* of 100 c.mm. of water is 9 c.mm.

But 100 c.mm. of water = 0.1 c.cm., and therefore weighs, approximately, 0.1 gram.

Hence, a contraction of 9 c.mm. indicates the melting of 0.1 gram of ice and the consequent absorption of $(0.1 \times 80) = 8$ gram-degree units of heat.

Therefore a contraction of 50 c.mm. indicates the absorption of

$$\frac{50 \times 8}{9} = 44 \frac{4}{9} \text{ gram-degree units of heat.}$$

4. If the specific heat of tin is 0.056, and the latent heat 14.25, what quantity of heat is required to raise 6 lbs. of tin from the temperature 208° C. to its melting point, 238° C., and to melt it?

7. Determine the latent heat of ice from the following data.

Weight of brass calorimeter (Sp. heat .09)...	...	35 gm.
" " " " + water	...	156 gm.
Initial temperature of water and calorimeter	...	24° C.
Final " "	...	17° C.
Weight of calorimeter, etc., after addition of ice	...	165 gm.

8. A gram of ice at 0° C. contracts 0.091 c.cm. in becoming water at 0° C. A piece of metal weighing 10 grams is heated to 50° C. and then dropped into the calorimeter. The total contraction is .063 c.cm. : find the specific heat of the metal, taking the latent heat of ice as 80.

9. Five hundred cubic centimetres of mercury at 56° C. are put into a hollow in a block of ice, and it is found that 159 grams of ice are liquefied ; find the specific heat of mercury.

10. Ten grams of water at 96° C. are placed in the inner tube of a Bunsen's calorimeter, and it is found that the volume of the contents of the outer portion decreases by 1.09 c.cm. : taking the latent heat of water as 80, what value does this give for the specific gravity of ice ?

CHAPTER VIII.

CHANGE OF STATE.

VAPORIZATION AND CONDENSATION: PROPERTIES OF VAPOURS.

54. THE passage of any substance from the solid to the liquid state takes place at a definite temperature, which varies but slightly with varying conditions of pressure. On the other hand, the passage of a substance from the liquid to the gaseous state takes place at all temperatures by the process of **evaporation**, and at a fixed temperature (for a given pressure) by the process of **ebullition**. This fixed temperature varies with the pressure, and, for any given pressure, is called the **boiling point** of the liquid at that pressure; when the pressure is that due to 760 mm. of mercury the corresponding temperature of ebullition is known as *the* boiling point of the liquid, and is one of the specific constants of the liquid.

55. **Properties of vapours.** Before discussing the laws of vaporization it will be necessary to consider the properties of vapours. When a liquid is converted into the gaseous state it becomes a vapour, and, as such, possesses properties similar to those of a gas. At temperatures not far above that of vaporization, vapours only approximately conform to the gaseous laws of Art. 32, but as the temperature rises they conform more and more closely to these laws, and ultimately behave like ordinary gases. A vapour thus becomes a gas at temperatures considerably above that of vaporization, and, in fact, all gases are merely vapours at temperatures far above the boiling point of the corresponding liquid.

56. Vapour pressure.* This subject is essentially experimental, and is therefore best considered by discussing illustrative experiments.

Let a large barometer tube be carefully filled with mercury, and inverted in a cistern containing a quantity of the same liquid. The tube now acts as a simple barometer; a column of mercury about 760 mm. in height remains in it, and above this column is the Torricellian vacuum. Into this vacuum we wish to introduce a small quantity of liquid; this can be done by passing a curved pipette, containing some of the liquid, under the mouth of the tube below the surface of the mercury in the cistern, and causing a small quantity to ascend through the column into the vacuum. Immediately on reaching the vacuum the liquid (if the quantity be not too large) at once disappears, and the column of mercury is depressed. The explanation of this is obvious: the liquid on entering the vacuum is immediately vaporized, and the vapour produced exerts a pressure, which causes the column of mercury to fall. As more liquid is introduced, drop by drop, it is quickly converted into vapour, and the mercury column gradually falls until, ultimately, a point is reached, at which no further change takes place; the drop of liquid rises through the column, but remains at the top as a thin layer of liquid, and there is no further fall of the mercury column. If this experiment be performed with different liquids it will be found that the course of the experiment is exactly the same for each liquid, but that the final pressures exerted by the vapours are widely different. For example, if the temperature of the experiments be 20°C. , the fall of the mercury column for water vapour is about 17 mm.; for alcohol vapour about 60 mm.; and for ether vapour about 400 mm. So far the experiment indicates that liquid in a vacuum is quickly converted into vapour, but that there is some limit to the quantity of liquid that can be so converted; also that the pressure exerted by this vapour gradually increases until it reaches a maximum value measured by the fall of the mercury column.

* This is sometimes called *vapour tension*, and though the term is inappropriate, custom has, to some extent, sanctioned its use.

We have now to find on what this limit depends, and also on what the maximum value of the pressure exerted by the vapour depends. Let us consider the influence of the volume of the vacuum and the temperature of the experiment.

1. *The influence of the volume of the vacuum.* Let several barometer tubes of *different* lengths be filled with mercury and inverted in a mercury cistern. The height of the column in each will be equal, and, as the tubes are of different lengths, the volume of the Torricellian vacuum in each will be different. If the experiment described above be repeated with each of these it will be found that the quantity of liquid which can be introduced into each tube, so as to be completely converted into vapour in the vacuum, depends on the volume of the vacuum, and is, for a given temperature, directly proportional to the volume of the space occupied by the vapour when the limit, referred to above, is reached. It will, however, be found that the maximum pressure exerted by the same vapour in each tube is exactly the same.

2. *The influence of the temperature at which vaporization takes place.* To investigate the influence of temperature on the formation of vapour we must somewhat modify our apparatus. The upper half of the barometer tube is surrounded by a wider tube closed below by an india-rubber stopper, through which the barometer tube passes. Into this wider tube, water, at different temperatures, can be poured, and thus the temperature at which vapour is formed in the vacuum can be varied. On carrying out the experiment, as detailed above, for a series of different temperatures it will be found that the maximum pressure exerted by the vapour increases very rapidly with the temperature; for example, the maximum pressure of water vapour at—

10° C.	is that due to	9.165 mm.	of mercury.		
20° C.	"	"	17.391	"	"
50° C.	"	"	91.981	"	"
70° C.	"	"	233.093	"	"

The quantity of liquid required to supply sufficient vapour to exert the maximum pressure will also be found to vary

with the temperature of vaporization, but as this vaporization is dependent on the maximum pressure at each temperature, it is sufficient to determine this pressure experimentally.

We have now established—

(i) That liquid is rapidly converted into vapour when placed in a vacuum.

(ii) That the quantity of liquid that can be so converted varies with the volume finally occupied by the vapour and with the temperature at which the conversion takes place.

(iii) That a vapour in a closed space, at a given temperature, exerts its maximum pressure only when a sufficient quantity of liquid has been introduced into that space, and that, when this point is reached, no more vapour can be produced. The space is then said to be *saturated* with vapour.

(iv) That the *maximum* pressure exerted by the vapour of a given liquid depends only on its temperature.

The last two results are the most important.

Maximum pressure must be understood in the sense explained above, and may be defined, for any temperature, as the pressure exerted, at that temperature, by a vapour when in a closed space *in contact with its liquid*.

So far we have considered the production of vapour in a vacuum; it remains now to find out how the results we have arrived at must be modified when vaporization takes place in a closed space containing air or other gas or vapour.

For this purpose the apparatus represented in Fig. 37 is most convenient.

A, B, and C are barometer tubes standing in a mercury cistern, consisting of a cylindrical iron vessel, screwed on to a piece of iron tubing closed at the lower end.

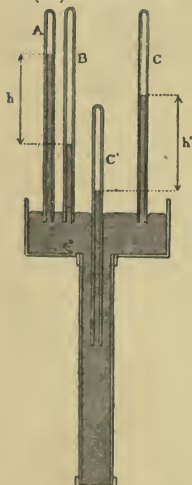


Fig. 37.

The whole apparatus is firmly supported by suitable stands. The tube A is a simple barometer tube, and the height of the column in it gives the barometric height. Into the tube B has been introduced a quantity of liquid sufficient to saturate the space above the mercury column; hence, the difference of height, h , between the column in A and that in B determines the maximum pressure exerted by the vapour in B at the temperature of the apparatus. Into C is first introduced a quantity of air sufficient to lower the column of mercury by about 300 mm.; the position of the top of the mercury column after the introduction of the air is then marked, by the edge of a piece of gummed paper fastened round the tube. A sufficient quantity of the liquid, previously used in the case of B, is now introduced into the space above the mercury column in C. This liquid, now in the presence of air, *gradually* passes into vapour, and the mercury column is slowly depressed and finally becomes stationary, indicating that the vapour is exerting its maximum pressure. To determine this maximum pressure, it is necessary to eliminate the pressure of the air in the tube; this can be done by lowering C into the tubular part of the cistern (as at C') until the level of the mercury column again coincides with the edge of the gummed paper. The air in the tube will now occupy the same volume as it did before the introduction of the liquid, and will therefore exert the same pressure, hence, the difference, h' , in the height of the column in C *above the surface of the mercury in the cistern* before and after the introduction of the liquid and the subsequent lowering in the cistern, determines the maximum pressure exerted by the vapour in the presence of the air. It is found, in this way, that the maximum pressure exerted by a particular vapour, at a given temperature, is the same, whether the vaporization takes place in the presence of air, or in a vacuum, that is, h' is found to be equal to h . The process, however, goes on much more slowly in the presence of air than in a vacuum. By substituting other gases and vapours for air it is found that the above result is true generally, and applies when several gases and vapours exist in the same space, except when they act chemically on one

another. It is also found that the quantity of vapour required to saturate a given space is the same, whether that space be a vacuum or occupied by other gases or vapours, having no chemical affinity for the vapour considered.

These questions relative to the formation of vapour were first studied by Dalton, and the more important of the results he arrived at have been formulated as **Dalton's Laws**. These are :—

(i) The maximum pressure exerted by a particular vapour, in a closed space, at a given temperature, depends only on that temperature, and is independent of the presence of other vapours or gases having no chemical affinity for it.

(ii) When several vapours and gases, having no chemical affinity for one another, are present in the same space, the actual pressure exerted by the mixture is the sum of the pressures exerted by the constituents of that mixture.

The first of these laws deals with maximum pressure, and is therefore applicable, only when the space is saturated with the vapour considered; the second law is applicable whether the space is saturated or not.

57. Pressure of aqueous vapour. The most important quantitative measurements relating to vapour pressure are those made by Dalton and Regnault in connection with aqueous vapour.

Dalton employed the simple method already referred to in the preceding article. His apparatus is shown in Fig. 38. C is a cast-iron cistern, containing mercury, in which are inverted two barometer tubes, T and T'. The tube T acts as an ordinary barometer, but a small quantity of water has been introduced into the vacuum in T'. Both tubes are surrounded by the large glass cylinder V V, which is nearly filled with water, resting on the surface of the mercury in the cistern C. The several parts of the apparatus are supported

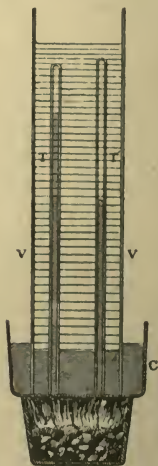


Fig. 38.

by suitable stands, and the whole arrangement rests on the furnace F. By regulating the heating effects of the furnace the temperature of the water in V V can be varied as required, and, by observing the difference of level in T and T', for any given temperature, the maximum vapour pressure for that temperature is determined.

TABLE OF PRESSURE OF AQUEOUS VAPOUR.

[Regnault.]

Temperature.	Pressure.	Temperature.	Pressure.
	mm.		mm.
-32° C.	0.320	90° C.	525.45
-20° C.	0.927	100° C.	760.00
-10° C.	2.093	110° C.	1075.37
-5° C.	3.113	120° C.	1491.28
0° C.	4.600	130° C.	2030.28
5° C.	6.534	140° C.	2717.63
10° C.	9.165	150° C.	3581.23
15° C.	12.699	160° C.	4651.62
20° C.	17.391	170° C.	5961.66
30° C.	31.548	180° C.	7546.39
40° C.	54.906	190° C.	9442.70
50° C.	91.982	200° C.	11688.96
60° C.	148.791	210° C.	14321.80
70° C.	233.093	220° C.	17390.36
80° C.	354.643	230° C.	20926.40

atmospheric

58. Saturated and unsaturated vapours. When a space is saturated with vapour, at a given temperature, this vapour exerts its maximum pressure for that temperature, and is also at its maximum density for that temperature, because the greatest possible quantity of vapour has been introduced into the space. Vapour in this condition is sometimes called **saturated vapour**. When vapour, at a given temperature, exerts a pressure less than its maximum pressure for that temperature, it is said to be **unsaturated**.

Vapour in contact with its liquid, in a closed space, is always saturated, for, the presence of the liquid is a proof that the limit of vaporization, under the existing conditions, has been reached.

The behaviour of saturated vapour under change of pressure and temperature should be considered. **Saturated vapour**, at any temperature, exerts the *maximum* pressure corresponding to the temperature, and hence any attempt to *increase its pressure*, e.g. by compressing it in a closed tube, must necessarily fail; the only effect produced by such an experiment would be the condensation of part of the vapour in the space, because, the vapour being already at its maximum density, for the given temperature, any attempt at further compression causes it to pass gradually into the liquid state, while its pressure remains constant. On the other hand, if saturated vapour be allowed to expand *in the presence of its liquid*, its pressure remains constant, and the liquid present is gradually converted into vapour, because, directly any small increase of volume takes place, the pressure of the vapour, in accordance with Boyle's law, tends to fall below the maximum pressure corresponding to the conditions of the experiment, and hence vaporization of the liquid present becomes possible, and thus, the pressure is maintained constant. If, however, there is no liquid present, the pressure of the vapour decreases on expansion in approximate agreement with Boyle's law, and the vapour becomes unsaturated.

The influence of change of temperature on a saturated vapour also depends on whether its liquid is present or not. If vapour be heated, in a closed space, in contact with its liquid, the liquid is converted into vapour, and the pressure increases at such a rate that, at any given temperature, the vapour present in the space saturates it, and the pressure exerted is the maximum pressure for that temperature. Similarly, if saturated vapour be cooled in a closed space, in contact with its liquid, the vapour condenses and the pressure decreases in such a way that, at any given temperature, the quantity of vapour present in the space is just sufficient to saturate it, and the pressure of the vapour is the maximum pressure for the given temperature.

If, however, saturated vapour be heated, in the absence of its liquid, it expands in accordance with Charles' law, or, if the heating take place at constant volume, its pressure

increases in the same way as that of an ordinary gas (Art. 32). On the other hand, if a saturated vapour be cooled in the absence of its liquid condensation at once takes place, and the vapour remains saturated.

Unsaturated vapours behave as ordinary gases under change of pressure and temperature, that is, they obey both Boyle's and Charles' laws except in two cases. To understand these two cases it should be remembered that, for any given vapour, there is, for each temperature at which it can exist, a definite maximum pressure corresponding to that temperature, and conversely, for any given pressure to which a vapour is subjected, there is a temperature at which this pressure is the maximum. Hence, if an unsaturated vapour, at a given temperature, be subjected to increase of pressure, its volume decreases in approximate accordance with Boyle's law, until a pressure is reached, which is the maximum pressure for the given temperature, and therefore, at this pressure the vapour becomes saturated, and further increase of pressure produces condensation, as described above. Again if an unsaturated vapour be cooled, at constant pressure, its volume will decrease in approximate accordance with Charles' law until a temperature is reached, for which the given pressure is the maximum, and therefore, at this temperature the vapour becomes saturated, and further cooling produces condensation.

60. Evaporation. We have seen that, at all temperatures, vaporization takes place in a closed space until that space is saturated with vapour. If, however, the process goes on in the open air, it becomes continuous, because it is practically impossible to saturate the atmosphere with vapour. This process of vaporization in an unlimited atmosphere has been called **evaporation**.

Evaporation takes place exclusively at the free surface of the liquid; the vapour formed at the surface penetrates into the adjacent layer of air, and from thence it passes into the next layer, and so on, until the vapour is diffused throughout the atmosphere, at a uniform rate, such, that any given layer of air passes on as much vapour as it

receives. This process is somewhat modified if the air is in motion, for then the layer in contact with the liquid is continually being renewed, and evaporation takes place more rapidly, because each layer, as it comes in contact with the liquid, takes up only a small quantity of vapour, which is carried away, and thus diffusion is assisted; moreover, evaporation always takes place into a comparatively dry atmosphere, for the successive layers pass on before becoming nearly saturated.

The evaporation of water is a process familiar to all. If a shallow vessel full of water be placed in the open air evaporation takes place, and in the course of a few hours all the water will have disappeared; the drying of clothes in the air is another instance of evaporation, as is also the drying up of rivers and pools of water during long periods of drought.

61. Conditions favourable to evaporation. Evaporation is favoured by any circumstance which facilitates the formation of vapour at the free surface of the liquid. From the laws of vaporization as explained in this chapter it may be deduced that the rate at which evaporation takes place is favoured by—

1. Dryness of the air.
2. Low atmospheric pressure.
3. High temperature, both of the liquid and of the air.
4. Large extent of free surface of liquid.
5. Low boiling point of liquid.
6. Renewal of air in contact with surface of liquid.

62. Ebullition. Consider a bubble of air in a liquid; its walls are formed by the liquid, and from these walls evaporation goes on until the space which they enclose becomes saturated with vapour. Let V denote the initial volume of air in the bubble, and P the hydrostatic pressure to which it is subjected. The value of P is determined by the position of the bubble in the liquid; if it is at a depth d below the surface, then P is equal to the atmospheric pressure, *plus* the pressure due to a column of the liquid of height d . As evaporation goes on into the bubble, the

pressure exerted by the vapour increases, until it becomes equal to F , the maximum vapour pressure corresponding to the temperature of the liquid. But, as the total pressure in the bubble must, so long as the bubble remains in the same position, be equal to P , the pressure of the air in it must, according to Dalton's second law, be equal to $(P - F)$.* Hence, if V' denote the volume of the air after the bubble is saturated with vapour, we have, by Boyle's law—

$$V P = V' (P - F), \text{ or, } V' = \frac{V P}{P - F}.$$

From this it is evident that V' increases as $(P - F)$ diminishes, and becomes infinitely great when $P - F = 0$, that is, when $F = P$. This means that when the temperature at any point in any liquid is such, that the maximum vapour pressure for that temperature is equal to the hydrostatic pressure at that point, then if a bubble of air, gas, or vapour form at that point its volume will tend to become infinitely great, that is, it will give rise to an infinite number of bubbles charged with vapour, which rise through the liquid, and escape at its surface. This process of formation of vapour in the interior of a liquid, and its final escape at the surface of the liquid, is called *boiling* or *ebullition*.

If a liquid be placed in an ordinary glass vessel and gradually heated, the temperature rises, and evaporation goes on freely at the surface of the liquid. Before long, however, small bubbles of air are seen to form on the walls of the vessel, and from them minute bubbles are given off, and rise to the surface; as the temperature rises, this process goes on more rapidly, and bubbles are given off freely from the different points on the walls of the vessel, where air bubbles have been seen to form. If the heat is applied from below, the upper layers of the liquid will be colder than the lower ones, and it will be noticed, at a certain stage of the heating, that bubbles given off from points near the bottom of the vessel are condensed in the upper colder layers before reaching the surface. The condensation of

* Let P' denote the pressure due to the air in the bubble, and F that due to the aqueous vapour, then applying Dalton's second law, $P' + F = \text{total pressure in the bubble, that is, } P' + F = P$, or, $P' = P - F$.

these bubbles of vapour is accompanied by a peculiar noise familiarly known as *singing*. On further heating the temperature of the liquid becomes more uniform, and large bubbles of vapour rise rapidly to the surface and escape. The liquid is thus kept in a state of agitation known as *boiling*.

It thus appears that the commencement of ebullition in a liquid is dependent on the presence of bubbles of air, or other gas, in the liquid or on the walls of the vessel in which it is heated. Experiment shows this to be the case; water, for example, can be heated up to 180°C . without entering into ebullition, provided no bubbles of air are allowed to form in it; but if, in any way, a bubble of vapour or any gas is introduced, boiling at once takes place with explosive violence.

On the other hand, the presence of a very minute quantity of air or other gas in a liquid is sufficient to produce continuous boiling for an indefinite time. This can be shown by dropping a small glass bulb, containing air, into a liquid at its boiling point; the air in the bulb will give rise to thousands of vapour bubbles without being appreciably diminished in volume.

63. Laws of ebullition. The following are the laws of ebullition—

1. At any given pressure, ebullition commences at a temperature,* which is fixed for each liquid. The temperature of ebullition at the normal atmospheric pressure is known as *the boiling point* of the liquid.

2. At any given pressure, ebullition commences at a temperature, such that the corresponding maximum vapour pressure is equal to the given pressure. Thus, a liquid boils at the normal atmospheric pressure, when its temperature is such, that the maximum pressure of its vapour, for that temperature, is equal to that due to 760 mm. of mercury.

3. The temperature remains constant during ebullition, so long as the pressure remains constant.

* Temperature, in each of these laws must be understood to mean temperature of the *vapour* produced. The temperature of the *liquid* is found to vary slightly with the nature of the surface of the vessel in which it is boiled.

64. Influence of pressure on the boiling point. It will be seen, from the laws given above, that the temperature, at which ebullition commences, is determined by the pressure to which the liquid is subjected. The second law defines the relation existing between pressure and the temperature of ebullition; and shows that, as external pressure increases, the temperature of ebullition rises, in such a way that the corresponding maximum vapour pressure always equals the external pressure.

These facts may be illustrated by some simple experiments.

1. Franklin's experiment. This experiment shows that, by diminishing the pressure at the surface of water, it can



Fig. 41.

be made to boil at temperatures considerably below 100°C . Water is boiled in a flask until all air is expelled, and its place taken by water vapour. The flask is then removed from the source of heat, tightly corked, inverted, and placed as in Fig. 41. It is now allowed to cool until ebullition entirely ceases; when this is the case, some cold water is poured over the vessel, so that the vapour inside is condensed, and the water again begins to boil, and continues to do so for some time. This is explained by the fact, that, when the vapour inside the flask is condensed, the pressure on the surface of the water is diminished, and when it becomes equal to the maximum pressure of the aqueous vapour for the temperature of the water, ebullition at once sets in.

2. The following experiment illustrates the fact, that, during ebullition, the maximum vapour pressure corresponding to the temperature of ebullition is equal to the external pressure. A glass tube is bent, as shown at A (Fig. 42), and the shorter limb is closed, the longer one being left open at the top. Mercury is introduced into the shorter limb, so as to entirely fill it; a small quantity of water is then passed up, and, displacing the mercury, occupies the upper portion of the limb. In the longer limb the mercury stands a little higher than the bend (Fig. 42, A). Water is now boiled in a wide-necked flask, and during ebullition the bent tube A is introduced into it so that the water in the short limb is surrounded by steam (Fig. 42, B); in a short time this water

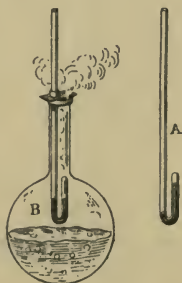


Fig. 42.

will be converted into steam, and if the space remains saturated, it will ultimately exert the maximum pressure corresponding to the temperature of the surrounding steam. It will be found that, when this is the case, the mercury assumes the same level in both branches of the tube, showing, that the maximum pressure for aqueous vapour, at the temperature of ebullition, is equal to the external pressure at which ebullition takes place.

3. If a small boiler be placed in communication with a reservoir of air, in which the pressure can be varied, it is found that the temperature of ebullition rises and falls with the pressure, and, if the latter be indicated by means of a manometer, attached to the reservoir, it is further found

that the pressure at which ebullition takes place, for any given temperature, is equal to the maximum vapour pressure for that temperature. Regnault adopted an elaborate form of this apparatus for determining the maximum pressure of aqueous vapour at temperatures above 50°C . He employed a small copper boiler, B, communicating, as shown in the figure (Fig. 43), through a condenser C with a reservoir of air R. Thermometers in the boiler gave the temperature of the vapour, and the pressure of the air in the reservoir was given by an attached mercury manometer M. In order

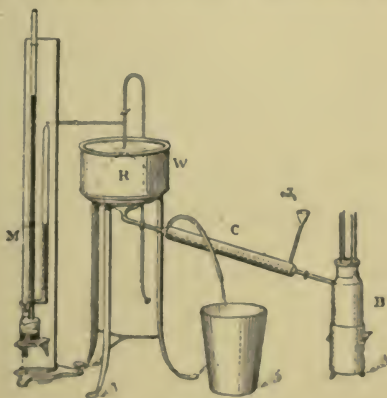


Fig. 43.

to protect the air in R from change of pressure due to variation of temperature it was surrounded with water in a bath W, and thus maintained at constant temperature. The condenser merely consisted of a rather long slanting tube placed between the boiler and the reservoir of air, and surrounded by a wider tube, through which a current of cold water constantly flowed. By varying the pressure in the reservoir the water in the boiler was made to boil at different temperatures, and by noting the pressure corresponding to each temperature, the maximum pressure of aqueous vapour was determined for a great number of

different temperatures. The condenser is a necessary part of the apparatus, because a liquid will boil in a closed vessel only when the space above it is sufficiently large and cool to admit of condensation of the vapour produced. In a closed vessel, where there is no provision for condensation of the vapour produced, water may be raised to very high temperatures. This fact is often of great utility in the industrial arts. An instance of its application is found in **Papin's digester**, which is used for the purpose of subjecting substances to the action of water at a temperature considerably higher than 100°C . It consists of a strong bronze vessel, covered with a lid secured by a screw and fitted with a safety valve, to prevent the internal pressure becoming greater than the vessel can sustain.



Fig. 44.

65. The hypsometer. This instrument (Fig. 44) is merely a small vessel conveniently constructed for observing the temperature of ebullition at any place. It consists of a small boiler heated by a spirit lamp, and is provided with a delicate thermometer which is surrounded by steam in the upper part of the vessel. The steam escapes at the top by a small side tube which communicates with the outer air, so that ebullition takes place under the atmospheric pressure. The hypsometer is used for the determination of heights, and is very convenient for this purpose because of its portability. The principle of its use for such determinations is based on the fact, that the temperature of ebullition depends on the external pressure, and that when the former is known the latter is determined, and is equal to the maximum vapour pressure at that temperature; hence, if we find the temperature at which water boils at the sea-level, and at a place the height of which we wish to determine, we can, by reference to a table of maximum pressures of aqueous vapour, find the corresponding atmospheric pressures, and when these are known, the required

height can be calculated. It is found that the boiling point of water falls 1°C. for every 1,080 feet increase in elevation. According to Soret's formula, the height (h) of any place above the sea-level is approximately given by $h = 295 (100 - t)$ where t is the temperature at which water boils at that place, and h the required height in *metres*. The fact that water boils below 100°C. at high elevations, is a source of inconvenience when cooking has to be done in places high above the sea-level. In such cases a modified form of Papin's digester is used.

66. Determination of the boiling point of a liquid. For this purpose it is only necessary to employ some simple form of apparatus similar to the hypsometer. The thermometer is placed in the vapour of the liquid, and the stationary reading, given after some minutes' continuous boiling, is taken as the boiling point of the liquid, at the pressure indicated at the time of the experiment, by the height of the barometer.

TABLE OF BOILING POINTS AT 760 MM. PRESSURE.

Nitrous oxide . . .	-87.9°C.	Benzene . . .	80.4°C.
Carbonic anhydride .	-78.2	Distilled water . .	100.0
Ammonia . . .	-33.5	Spirits of turpentine .	130.0
Chlorine . . .	-33.6	Oil of turpentine . .	159.1
Sulphurous anhydride	-10.0	Phosphorus . . .	290.0
Ether . . .	35.0	Linseed oil . . .	316.0
Sulphide of carbon .	46.2	Sulphuric acid . . .	325.0
Chloroform . . .	60.2	Mercury . . .	357.0
Alcohol . . .	78.3	Sulphur . . .	440.0

67. Latent heat of vaporization. We have already considered the question of latent heat in some detail. It should be remembered that whenever liquid is converted into vapour, whether during evaporation or ebullition, heat is absorbed in the process, and the amount of heat absorbed is equivalent to the work done in effecting the change of

state. During evaporation* this absorption of heat greatly cools the liquid, and, under certain conditions, may cause it to freeze. Several experiments have been devised to illustrate this fact.

1. **Leslie's experiment.** A very small shallow pan, containing a little water, is placed, together with a vessel of sulphuric acid, under the receiver of an air-pump. The water rapidly evaporates, and the vapour formed is absorbed by the sulphuric acid; this ensures continuous and rapid evaporation of the water. The heat absorbed during the process ultimately causes freezing of the water, and a thin layer of ice appears on its surface.

2. **The Cryophorus.** Wollaston's cryophorus (Fig. 45) consists of a bent tube with a bulb at each end. It contains only a small quantity of water and aqueous vapour, all air having been expelled, by boiling the water in one bulb and hermetically sealing the other when the bulb was full of steam. To illustrate the absorption of heat during rapid evaporation all the water is passed into the bulb B (Fig. 45), and the bulb A is surrounded, as shown

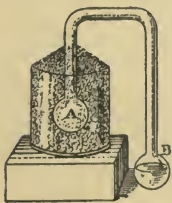


Fig. 45.

in the figure, with melting ice. The cold condenses the vapour in A as rapidly as it is formed in B, and, in a short time, the absorption of heat is sufficient to cool the water in B down to the freezing point, and finally convert it into ice.

3. A small quantity of water in a test tube may be frozen by placing it in a beaker containing ether, through which a current of air is rapidly passed, so as to promote its evapora-

* During evaporation the heat necessary to effect the required change of state is absorbed from the liquid itself. The temperature of the liquid is thus lowered, but, as evaporation can take place at any temperature, the process goes on, and a continuous fall of temperature is thus produced. With ebullition, however, the case is very different; the process can go on only at the temperature corresponding to the external pressure, and hence, unless heat is continuously supplied from without, the liquid cools below this temperature and ebullition ceases. Latent heat is generally considered in connection with the change of state at constant temperature.

ation. During evaporation, the ether rapidly absorbs heat from the water in the test tube, and in a few minutes ice begins to form.

Several somewhat complicated forms of apparatus have been constructed for freezing water on the principle illustrated by the above experiments; among these are Carré's apparatus for freezing by sulphuric acid, and also Carré's ammoniacal apparatus.

Another instance of the application of the cold produced by evaporation is the method adopted in India for keeping drinking water cool. The water is placed in porous earthenware vessels, through which it slowly percolates; evaporation takes place from the outer surface of the vessel, and the absorption of heat thus produced keeps the water in the desired state of coolness. The same effect may be produced in any vessel by wrapping a wet cloth round it.

68. Determination of the latent heat of steam. The latent heat of vapour, at any temperature, is defined, as the quantity of heat required to convert unit mass of the given liquid, at that temperature, into vapour at the same temperature. When we speak of *the* latent heat of any vapour, we generally mean the latent heat of the vapour at *the* boiling point of the corresponding liquid, but, in strictness, we ought always to speak of the latent heat at some particular temperature. Thus, by the latent heat of steam we usually mean the latent heat at $100^{\circ}\text{C}.$, that is, the quantity of heat required to convert unit mass of water, at $100^{\circ}\text{C}.$, into steam at $100^{\circ}\text{C}.$, or, in other words, the quantity of heat given out by unit mass of steam, at $100^{\circ}\text{C}.$, in becoming water at $100^{\circ}\text{C}.$

To determine the latent heat of steam at $100^{\circ}\text{C}.$ the usual calorimetric apparatus is required, together with a small boiler for producing steam, and an arrangement for *drying* the steam so produced. A convenient apparatus is a small copper boiler, containing a spirally coiled copper tube, through which the steam produced is made to pass before issuing from the boiler. It is thus dried, that is, the small particles of water with which it was mixed when first

formed, are completely converted into steam; in this condition it is passed, through a short communicating tube, into a known weight of water at a known temperature in the calorimeter. On entering the water the steam is immediately condensed, and, giving out its latent heat, raises the temperature of the water. When a sufficient rise of temperature is produced, the steam is shut off, the final temperature noted, and the calorimeter is then weighed in order to determine, from its increase of weight, the quantity of steam which has been converted into water.

Let M denote the mass of water in the calorimeter.

„	w	„	„	water equivalent of the calorimeter.
„	t	„	„	initial temperature of the water in the calorimeter.
„	θ	„	„	final temperature of the water in the calorimeter.
„	T	„	„	temperature of the steam.
„	m	„	„	mass of steam converted into water.
„	L	„	„	latent heat of steam.

Then, for loss of heat we have—

(a) By steam during change of state from steam, at T° C. to water at T° C. = $m L$.

(b) By cooling of this water from T° C. to θ° C. = $m(T - \theta)$.

Also, for gain of heat we have—

(a) By water in the calorimeter = $M(\theta - t)$

(b) By calorimeter = $w(\theta - t)$

On equating the loss and gain of heat we get—

$$m L + m (T - \theta) = (M + w) (\theta - t),$$

or—

$$L = \frac{(M + w)(\theta - t)}{m} - (T - \theta);$$

and thus the latent heat at T° C. is determined.

The calorimeter employed in this experiment should be larger than that used in specific heat determinations in order to prevent the steam, on entering the calorimeter, from flowing directly on to the thermometer bulb, and thus raising its temperature more than that of the surrounding

water. The rise of temperature produced by the introduction of the steam should be from 10°C. to 20°C. , and to prevent loss of heat by cooling it is well to arrange, that, at the commencement of the experiment, t is as much below the temperature of the room as θ is above it at the end of the experiment.

A very rough determination of the latent heat of steam may be made by passing steam from water boiling in a flask, directly into the calorimeter. The sources of error in such an experiment are so many and so great that the results obtained often vary by from 10 to 20 per cent. from the true result. A better form of apparatus for rough determinations is that shown in Fig. 46.

The flask F is about half filled with water, fitted with a cork carrying the tube tt , and fixed in an inverted position by means of a suitable clamp on a retort stand. The water is heated by means of a simple ring burner BB , and the steam passes down the tube tt where it is dried. SS is a screen, supported on a ring on the retort stand, to shield the calorimeter from the heat of the burner B and the water in the flask. Attached to the end of the tube tt , by means of a short piece of india-rubber tubing, i , is a nozzle n , which is a thin glass tube, obtained by drawing out a piece of thick tubing. This nozzle dips into the water in the calorimeter C , and the current of steam can be cut off, for an instant, by pinching the india-rubber tube i , which should be surrounded with cotton wool. To perform the experiment, steam is allowed to pass from the nozzle for some time; the tube i is then pinched, for an instant, and the calorimeter quickly put in position. The steam is then allowed to pass, until the required rise of temperature is produced; when this is the case, the calorimeter is lowered, until the mouth of the nozzle is just above the surface of the water, the current of steam is then stopped, while the calorimeter is withdrawn. The reading of the thermometer

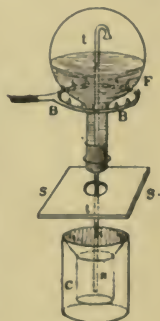


Fig. 46.

should be noticed for a short time after the withdrawal of the calorimeter, and the maximum temperature attained should be taken as θ .

69. Regnault's results in connection with latent heat.

Regnault investigated the question of latent heat of steam in a series of careful and elaborate experiments, and found that its value depends upon the temperature at which vaporization takes place. From the data of a large number of experiments he was able to formulate his results thus: If Q denote the total heat required to raise unit mass of water from 0°C. to $T^\circ \text{C.}$, and also to convert it into steam at $T^\circ \text{C.}$, then—

$$Q = 606.5 + 0.305T. \quad (1)$$

But—

$$Q = L + T$$

for, L units of heat are absorbed during change of state and T units are absorbed by the water, in being raised from 0°C. to $T^\circ \text{C.}$

Thus we have—

$$L = Q - T;$$

or—

$$\begin{aligned} L &= 606.5 + 0.305T - T, \\ &= 606.5 - 0.695T. \end{aligned} \quad (2)$$

Hence, we have that the total heat required to raise unit mass of water from 0°C. to 100°C. , and to convert it into steam at 100° is given by—

$$\begin{aligned} Q_{100} &= 606.5 + (0.305) 100 \\ &= 606.5 + 30.5 \\ &= 637; \end{aligned}$$

and similarly—

$$\begin{aligned} L_{100} &= 606.5 - 69.5 \\ &= 537. \end{aligned}$$

That is, in order to convert unit mass of water, at 100°C. , into steam at 100°C. , 537 units of heat are required. This is generally expressed by saying that the latent heat of steam at 100°C. is 537 heat units.* These formulæ for Q and L also indicate the general result, that Q increases while L decreases with rise of temperature.

* See footnote, Art. 50.

We have hitherto spoken only of the variation of L with temperature, but it is evident that this implies variation with pressure; for, when we speak of the latent heat of steam at $t^{\circ}\text{C.}$ we imply that vaporization takes place at a pressure equal to the maximum pressure of aqueous vapour at $t^{\circ}\text{C.}$

TABLE OF LATENT HEATS OF VAPORIZATION.

Water	536.67	Ether	90.45
Wood spirit	263.70	Carbonic disulphide	86.67
Alcohol	202.40	Oil of turpentine	74.00
Acetic acid	101.91	Bromine	45.60
Acetic ether	105.80	Iodine	23.95

70. Condensation and distillation. We have already seen that condensation is the inverse of vaporization, and is accompanied by corresponding thermal effects. Distillation is a combined process of vaporization and condensation; it is much used for obtaining pure or distilled water from ordinary tap water, or from salt water. It is also of frequent use for manufacturing and chemical purposes. The liquid to be distilled is heated in a suitable vessel and the vapour produced passes through a long spirally coiled tube, surrounded by cold water, or is, in some other way, cooled to a temperature sufficiently low to cause the vapour to condense. The liquid thus produced is collected in a vessel into which the worm opens; it is free from all solid matter contained, either in suspension or solution, in the original liquid. It should be remembered in connection with this question that when a closed vessel, or system of communicating vessels, contains only a liquid and its vapour, at different temperatures, the pressure in the space occupied by the vapour is that determined by the *lowest temperature*, just as the strength of a chain or rope is determined by the strength of the *weakest point* (Art. 57). This explains why distillation is possible in a closed vessel (compare the cryophorus) or system of vessels; vapour is continually formed in the parts at higher temperature and is condensed where the temperature is lowest. The rate of condensation is such as to keep the pressure constant at the maximum vapour pressure corresponding to this lowest temperature.

CALCULATIONS.

71. In connection with the subject-matter of the preceding chapter the following points bearing on calculation should be noticed.

1. **Dalton's Second Law** (p. 110). If p_1, p_2, p_3 denote the individual pressures due to the several constituents of a mixture of vapours then, if this law be applicable, we have, $P = p_1 + p_2 + p_3$, where P is the total pressure exerted by the mixture. The most familiar example of the application of this law is in the case of air and water vapour. If P' denote the pressure due to the air alone, and f that due to the vapour, then the total pressure, P , is given by $P = P' + f$.

2. **Latent heat of vaporization.** If L denote the latent heat of vaporization of a given liquid then the quantity of **heat absorbed during vaporization** by a mass m of that liquid is denoted by mL , and the quantity of **heat evolved during condensation** of a mass m of the liquid is also denoted by mL . (Compare Art. 53, 1.)

Regnault's formulæ—

$$\begin{aligned} Q_T &= 606.5 + 0.305T, \\ L_T &= 606.5 - 0.695T, \end{aligned}$$

should be noted, and also the general result deduced on p. 126.

EXAMPLES IX.

1. A quantity of hydrogen is collected, over water, in a eudiometer tube. The height of the column of water, left in the tube, is 40.8 mm., and its temperature is 15° C. ; find the pressure of the hydrogen in the upper part of the tube. (Take the height of the barometer at 758 mm.) The space occupied by the hydrogen is saturated with aqueous vapour. Hence, with the notation used above—

$$\begin{aligned} P &= P' + f, \\ \therefore P' &= P - f, \end{aligned}$$

where P denotes the total pressure in the tube, P' that due to the hydrogen, and f that due to the aqueous vapour present.

But since 40.8 mm. of water are equivalent to $\frac{40.8}{13.6} = 3$ mm. of mercury we have that—

$$P = 758 - 3 = 755 \text{ mm.}$$

and, from table on page 111, we get—

$$\begin{aligned} f &= 12.7 \text{ mm.} \\ \therefore P' &= 755 - 12.7 = 742.3 \text{ mm.} \end{aligned}$$

EXAMINATION QUESTIONS.

QUESTIONS SET AT LONDON UNIVERSITY EXAMINATIONS.

Matriculation.

10. Define the specific heat and the latent heat of fusion of a substance.

The specific heat of iron is $\cdot 113$; how many lbs. of iron at 250° C. must be introduced into an ice calorimeter in order to produce 2 lbs. of water? *June, 1881.*

11. State the laws of evaporation. Under what circumstances will a liquid evaporate, and how must the conditions be modified in order that it may boil? What is the dew-point? *Ibid.*

12. Define specific heat. What is meant by the latent heat of steam? How does it vary with the temperature at which the steam is produced? *Jan., 1882.*

13. Twenty grams of iron at 98° C. (specific heat $\cdot 119$) are immersed in 80 grams of water at 10° C. contained in a copper vessel whose mass is 15 grams. Find the resulting temperature the specific heat of copper being $\cdot 095$.

What precautions should be taken to prevent loss of heat during the experiment? *Ibid.*

14. What do you understand by a unit of heat?

How would you determine experimentally the number of units of heat required to convert 1 lb. of water at 100° C. into steam at the same temperature? Describe fully the details of the experiment.

June, 1882.

15. Describe a method of determining the specific heat of a solid.

How many units of heat would cause a mixture of ice and water to contract by 50 c.mm., if 100 c.mm. of water at 0° C. become 109 c.mm. of ice on freezing? *Jan., 1883.*

16. What is meant by the statement that the latent heat of steam is 537? One pound of saturated steam at 160° C. is blown into 19 lbs. of water at 0° C., and the resulting temperature is $32\cdot 765^{\circ}$ C. Find the latent heat of steam at 160° C. *June, 1883.*

17. State clearly the distinction between temperature and heat.

Twenty pound-degrees of heat are communicated to a metal vessel weighing 8 lbs., and containing 10 lbs., of water. If the specific heat of the metal be $\frac{1}{10}$, in what proportion will the heat be divided between the water and the vessel, and what will be their rise of temperature? *Jan., 1884.*

18. Describe Bunsen's calorimeter. If 100 c.cm. of water in freezing become 109 c.cm. of ice, and the introduction of 20 grams of mercury at $100^{\circ}\text{C}.$, into a Bunsen's calorimeter cause the end of the column of mercury to move through 74 mm. in a tube 1 sq. mm. in section, find the specific heat of mercury. (The heat required to melt one gram of ice is 80 units.) *Jan., 1884.*

19. Explain fully the meaning of the statement that the latent heat of steam is 537. The specific heat of mercury is $\cdot 03$. A pound of steam at $100^{\circ}\text{C}.$ is made to pass into a vessel containing 300 lbs. of mercury initially at $0^{\circ}\text{C}.$, the capacity for heat of the vessel being equal to that of 10 lbs. of water. What will be the temperature of the vessel and contents at the end of the experiment? *June, 1884.*

20. 200 grams of water at $99^{\circ}\text{C}.$ are mixed with 200 c.cm. of milk of density $1\cdot 03$ at $15^{\circ}\text{C}.$, contained in a copper vessel of thermal capacity equal to that of 8 grams of water, and the temperature of the mixture is $57^{\circ}\text{C}.$ If all the heat lost by the water is gained by the milk and the copper, what is the specific heat of the milk? *Jan., 1885.*

21. A pound of ice at $0^{\circ}\text{C}.$ is thrown into 6 lbs. of water at $15^{\circ}\text{C}.$ contained in a copper vessel weighing 3 lbs. and when the ice is melted the temperature of the water is $2^{\circ}\text{C}.$ Find the latent heat of fusion of ice, the specific heat of copper being $0\cdot 095$. *June, 1886.*

22. Define specific heat and describe an experiment by means of which the specific heats of oil and water may be compared. *Jan., 1887.*

23. Explain the term *latent heat*. If 25 grams of steam at $100^{\circ}\text{C}.$ be passed into 300 grams of ice-cold water, what will be the temperature of the mixture, the latent heat of steam being taken equal to 536? *Ibid.*

24. Describe experiments illustrating the difference between temperature and heat. In 100 grams of boiling water ($t = 100^{\circ}$) there are placed 20 grams of ice and the temperature falls to 70° when the ice is just melted. What is the latent heat of fusion of ice, assuming no heat to be lost? *June, 1887.*

25. Distinguish between saturated and unsaturated vapour. What is meant by the statement, that when the dew-point is $20^{\circ}\text{C}.$, the maximum pressure of aqueous vapour in the air is that due to 17.4 mm. of mercury? *Ibid.*

26. Define the terms latent heat, specific heat, and capacity for heat. The specific heat of copper is $\cdot 095$. What is the capacity for heat of 500 grams of copper? If 500 grams of copper are heated to $100^{\circ}\text{C}.$, and placed in an ice calorimeter, how much ice is melted, the latent heat of fusion of ice being 80?

CHAPTER IX.

HYGROMETRY.

72. EVAPORATION is constantly going on from the surface of the sea, rivers, and lakes; and thus there is always more or less moisture in the air. We may convince ourselves of this by a few simple experiments.

(1) Fill a glass vessel full of ice-cold water and, after thoroughly drying the outer surface of the vessel, let it be exposed to the air; in general, owing to the condensation of the aqueous vapour in the air, beads of moisture will quickly form on the surface of the cold glass.

(2) Pass a current of air for some time through a U-tube containing pieces of calcium chloride or pumice-stone soaked in sulphuric acid. If the U-tube be weighed before and after the experiment it will be found to have increased in weight on account of the moisture absorbed from the air.

73. Relative humidity. Hygrometry deals with all measurements connected with the state of the air as regards the moisture present in it. We speak of the air as being *dry* or *moist* according as we think it contains little or much moisture, but the condition of the air, in relation to dryness or moisture, involves two elements: (1) the quantity of vapour *actually present* in the air; (2) the quantity of vapour necessary *to saturate* the air under the same conditions, and it is on the *ratio* of these two elements that our sensations of dryness and moisture chiefly depend and not on the first alone. Thus, the air in a warm room may really contain more aqueous vapour than the outside air and yet be *drier* because the amount required to saturate it is so much greater; for, we have seen that the mass of vapour required to saturate a given space increases with the temperature, and is independent of the presence of air or other gas in that space. *The hygrometric*

state of the air, considered as the ratio of the mass of aqueous vapour actually present in a given volume to the mass of vapour required to saturate the same volume, at the same temperature, has been called the humidity or relative humidity of the air, and is usually expressed either as a ratio or as a percentage. For example, if a cubic metre of air at a particular temperature contain 10 grams of aqueous vapour, and if 30 grams are required to saturate it at the same temperature, then the relative humidity is $\frac{1}{3}$, or 33·3, if expressed as a percentage.

75. Direct determination of relative humidity. In order to determine the relative humidity of the air, in the way explained above, it is evident that we must determine, by actual experiment, the mass of vapour really present in a given volume of air, and must also calculate the mass of vapour required to saturate the same volume at the same temperature. For this purpose a quantity of air, the humidity of which is to be determined, is drawn through a series of drying tubes containing fragments of pumice-stone soaked in concentrated sulphuric acid. To effect this the drying tubes are connected together by short communicating tubes, and one end of the series is connected to the upper tube of an aspirator,* while the other opens into the outer air. The aspirator is filled with water, and, when the communication with the drying tubes is effected, the upper and lower stopcocks are opened. The water then flows out by the lower tube and air is sucked in through the drying tubes, and is collected in the upper part of the aspirator. The air, in passing through the drying tubes, is deprived of its moisture, and by determining the increase in weight of the first two or three tubes, the weight of moisture deposited is known. The volume of the air drawn through the apparatus is determined by the weight of the water which has escaped from the aspirator. This volume

* The aspirator, Fig. 48, A, is a large vessel of from 50 to 100 litres capacity, having a tube, fitted with a stopcock at its upper and lower ends.

being determined, it only remains to calculate the mass of vapour necessary to saturate it at its initial temperature.

This method of determining relative humidity is known as the *chemical method*.

76. Dew-point. The chemical method, described above, leaves nothing to be desired so far as accuracy is concerned, but it is a somewhat tedious and difficult process, and, as it is very often necessary, for meteorological and other purposes, to know the hygrometric state of the air, simpler methods, based upon the determination of the *dew-point*, have been adopted. Imagine a given volume of moist air as isolated, by an imaginary envelope, from the surrounding air; if the temperature of this volume of air be gradually lowered, both the air and vapour will contract according to the same law, but the pressure of each will remain unchanged, the sum of the two pressures being, by Dalton's second law, always equal to the constant atmospheric pressure. As the temperature is lowered, a point is finally reached at which condensation of the vapour present begins; this evidently takes place at the temperature for which the existing vapour pressure is the maximum pressure, and this temperature is known as the **dew-point**, which may be defined, as *the temperature at which the vapour actually present in the air is sufficient to saturate it*. Hence, if we cool a mass of air down until condensation begins, at a temperature t , then, the maximum pressure of aqueous vapour at this temperature, gives the initial pressure of the vapour present in the air, for, since this pressure remains constant during the cooling, if we can determine its value at one stage of the process, we know that it must have been the same at any other stage. If, therefore, we know the dew-point, we can determine the pressure exerted by the aqueous vapour normally present in the air, and it can be shown that the relative humidity is determined by the ratio of this pressure to the maximum pressure of aqueous vapour at the original temperature of the air. That is, if h denote the relative humidity of

the air, f the maximum pressure of aqueous vapour at the dew-point, and F the maximum pressure at the temperature of the air, then—

$$h = \frac{f}{F}.$$

For example, if air, at 15°C. , be cooled down to 10°C. , before condensation commences, then—

$$h = \frac{f_{10}}{f_{15}} = \frac{9.2}{12.7} = .724; \text{ or } 72.4 \text{ per cent.}$$

77. Dew-point hygrometers. These are instruments devised for determining the dew-point, and thus, as explained above, indicating the hygrometric state of the air; they assume somewhat different forms, but the principle of action is the same in each. The more important forms are—

1. **Daniell's hygrometer.** This is a simple modification of the cryophorus, containing ether instead of water. One of the globes, A (Fig. 47), is made of black glass, or has a zone of gilding round it, and contains a small thermometer t ; the other, B , is a plain glass bulb. The tube and attached bulbs are mounted on a suitable stand carrying a thermometer, t' , which serves to indicate the temperature of the air. To use the instrument, all the ether is passed into A , and B is covered with muslin; a small quantity of ether is then poured on this muslin, from which it rapidly evaporates, and, in so doing, absorbs heat from B . The cold thus produced tends to condense the ether vapour in B ; this causes rapid evaporation from the surface of the liquid in A , and consequently, the temperature indicated by the thermometer t rapidly falls. As this goes on the surface of A is carefully watched, and the temperature, given by t , at which dew begins to form on it is carefully noted. The instrument is now left to itself until the dew



Fig. 47.

thus formed on A entirely disappears, and the reading of t at the instant of disappearance is recorded. The dew-point is very approximately given by the mean of the two readings thus taken.

The temperature of the air is given by the thermometer t' , and thus sufficient data for the determination of relative humidity are obtained.

2. Regnault's hygrometer. This is really an improved form of Daniell's hygrometer. It consists (Fig. 48) of a glass tube, T, closed below by a thin silver cap S. Its mouth is fitted with a cork carrying a thermometer t and a piece of tubing d , both passing down nearly to the bottom of the tube. To use the instrument ether or

alcohol is placed in the tube and a current of air is drawn through by the aspirator A; entering by d , the air bubbles through the liquid and enters A through the side tube e and the vertical tube V. This passage of air through the ether or alcohol in S causes rapid evaporation to take place; the temperature of

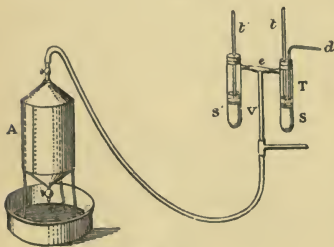


Fig. 48.

the liquid therefore falls, and ultimately the aqueous vapour in the air condenses on the surface of S.

As with Daniell's hygrometer, the dew-point is obtained by reading the thermometer t at the instants of appearance and disappearance of the film of moisture on the bright surface of S, and taking the mean of the two readings so obtained.

This form of the instrument has the advantage that, by regulating the current of air, the cooling of the liquid in S may be made sufficiently slow to note, with accuracy, the temperature at which the film of moisture commences to form on S. This observation is usually further facilitated by having a second tube, S' , similar to the hygrometer tube, supported on the same stand, in such a position that the

appearance of the two silver caps may be easily contrasted and the dimness due to the deposition of moisture on S more readily detected. S' is not, however, an essential part of the instrument; it has no connection with S, and serves only for comparison, as explained above, and to carry a thermometer, t' , which, like that on the stand of Daniell's hygrometer, gives the temperature of the air.

Regnault's hygrometer has several advantages over Daniell's form of the instrument; the most important of these are—

- (i) The rate of cooling can be more easily regulated.
- (ii) The observations are made by means of a telescope at some distance from the apparatus, so that all risk of error, due to the proximity of the observer, is obviated.
- (iii) The current of air passing through the liquid secures uniformity of temperature throughout its mass, and, as the silver caps are good conductors of heat and very thin, the temperature indicated by the thermometer in S is almost exactly the temperature of the surface of S at which condensation takes place.

In order to obtain good determinations with any form of dew-point hygrometer, the experiment should be so conducted that the temperatures of appearance and disappearance of the film of moisture, on the cold surface, are very nearly the same.

78. Wet and dry bulb hygrometer. This instrument is sometimes called *Mason's hygrometer*, and also *August's psychrometer*. It consists of two exactly similar thermometers, mounted on the same stand in such a way that the air can circulate freely round their bulbs. One thermometer is used merely to give the temperature of the air; the other has its bulb covered with muslin, which is kept moist by means of an attached wick communicating with a vessel of water. The evaporation which takes place from this wet bulb, produces a fall of temperature, and consequently the wet bulb thermometer always reads lower than the dry bulb. Now it is evident that this difference

of temperature between the two thermometers, depending, as it does, on the cold produced by evaporation from the wet bulb, will be proportional to the rate at which evaporation takes place, and, as this depends on the quantity of moisture already in the air, we can see that the difference of temperature of the thermometers is indirectly related to the hygrometric state of the air. If this relation can be discovered it may serve to determine the humidity of the air, but, as the assumptions involved are somewhat doubtful, it is usual to determine the dew-point from the thermometric readings, by means of tables, empirically constructed from the simultaneous indications of a dew-point instrument.

79. Hygrometers are contrivances with hair, cat-gut, etc., which serve to indicate the humidity of the air by means of some effect produced by the absorption of moisture. Many substances absorb moisture to a degree dependent on the quantity present in the air, and this absorption is generally accompanied by some variation in the form or condition of the substance. The nature of the variation depends on the structure; for instance, membranous bodies, such as paper, parchment, etc., expand and contract regularly according as they absorb or lose moisture; filamentous substances change much more in thickness than in length; ropes, strings, and threads, which are composed of twisted fibres, swell out and shorten* on the absorption of moisture. Hair, owing to its peculiar structure, shortens as the humidity increases. A single hair is made up of a series of cones fitting one into the other; on absorption of moisture, it swells out, causing the vertical angles of the cones to increase, and consequently they fit better into one another, and the hair contracts.

* [It should be noticed that the individual fibres of a rope, etc., probably increase *slightly* in length, but the rope, as a whole, shortens for the same reason that a number of thick strings form, on twisting, a shorter rope than the same number of fine strings of the same length twisted to the same extent.]

De Saussure's hygroscope (Fig. 50) is of historic interest, as being the first instrument devised for hygrometric purposes. It consists of a hair from which all grease has been removed by washing it in a solution of soda. It is fastened at one end to the screw-pin *s*, and at the other passes once round the pulley *p*, and carries the small weight *w*, which keeps it tightly stretched. Attached to the pulley is the needle *n*, which indicates, on a magnified scale, the change in length of the hair corresponding to changes in the humidity of the air. The scale round which the needle moves is graduated to indicate various degrees of humidity, but the indications are only relative, and the instrument is never used for scientific purposes.



Fig. 50.

Table showing pressure of aqueous vapour (*f*) in mm. and mass of water (*m*) in grams contained in 1 cubic metre of air, with dew-point *t*. (*Regnault and Magnus.*)

<i>t</i>	<i>f</i>	<i>m</i>	<i>t</i>	<i>f</i>	<i>m</i>	<i>t</i>	<i>f</i>	<i>m</i>	<i>t</i>	<i>f</i>	<i>m</i>
	mm.	gr.		mm.	gr.		mm.	gr.		mm.	gr.
-10°	2.0	2.1	0°	4.6	4.9	10°	9.1	9.4	20°	17.4	17.2
-9	2.2	2.4	1	4.9	5.2	11	9.8	10.0	21	18.5	18.2
-8	2.4	2.7	2	5.3	5.6	12	10.4	10.6	22	19.7	19.3
-7	2.6	3.0	3	5.7	6.0	13	11.1	11.3	23	20.9	20.4
-6	2.8	3.2	4	6.1	6.4	14	11.9	12.0	24	22.2	21.5
-5	3.1	3.5	5	6.5	6.8	15	12.7	12.8	25	23.6	22.9
-4	3.3	3.8	6	7.0	7.3	16	13.5	13.6	26	25.0	24.2
-3	3.6	4.1	7	7.5	7.7	17	14.4	14.5	27	26.5	25.6
-2	3.9	4.4	8	8.0	8.1	18	15.4	15.1	28	28.1	27.0
-1	4.2	4.6	9	8.5	8.8	19	16.3	16.2	29	29.8	28.6
0	4.6	4.9	10	9.1	9.4	20	17.4	17.2	30	31.6	30.1

CALCULATIONS.

80. THE principles necessary for the solution of problems connected with hygrometry have already been dealt with in Arts. 74—76 of the preceding chapter. The following points may be again noted :—

1. In calculating the mass of aqueous vapour present in a given volume of air it must be remembered that the total pressure of the mixture is made up of two pressures: (*a*) the pressure of the air; (*b*) the pressure of the vapour present. The latter pressure is equal to the maximum pressure of aqueous vapour at the dew-point, and is the pressure to be employed in calculating the required mass.

2. If the hygrometric state, or relative humidity of air be denoted by h , then—

$$h = \frac{m}{m'},$$

where m denotes the mass of aqueous vapour actually present in the air, and m' denotes the mass of aqueous vapour necessary to saturate the air under the existing conditions.

Also—

$$h = \frac{f}{F},$$

where f denotes the maximum pressure of aqueous vapour at the dew-point, and F denotes the maximum pressure of aqueous vapour corresponding to the temperature of the air.

3. The mass of 1 litre of dry air at 0° C. and 760 mm. pressure is 1.293 grams.

The mass of 1 litre of aqueous vapour at 0° C. and 760 mm. pressure is approximately $\frac{5}{8}(1.293)$ grams = 0.808 grams.

EXAMPLES X.

1. Two cubic metres of moist air, at 17° C., were drawn through a chemical hygrometer, and 24.12 grams of water were deposited in the tubes. Find the relative humidity of the air.

From above—

$$h = \frac{m}{m'}.$$

Here—

$$m = 24.12 \text{ grams,}$$

and m' denotes the mass of aqueous vapour necessary to saturate 2 cubic metres (*i.e.* 2,000 litres) at 17°C . The maximum pressure of aqueous vapour at 17°C . = 14.4 mm . (See Table on p. 139.)

$$\therefore m' = \frac{2,000 \times 0.808 \times 14.4 \times 273}{760 \times 290}$$

$$= 28.84 \text{ grams.}$$

$$\therefore h = \frac{m}{m'} = \frac{24.12}{28.84} = 0.837 \text{ nearly ;}$$

or, the percentage humidity = 83.7 .

3. Find the hygrometric state of air at 20°C ., the dew-point being 5°C .

The maximum pressure of aqueous vapour at—

$$5^\circ \text{C.} = 6.5 \text{ mm.} = f,$$

$$20^\circ \text{C} = 17.4 \text{ mm.} = F,$$

$$\therefore h = \frac{f}{F} = \frac{6.5}{17.4} = .374 ;$$

or, as a percentage—

$$h = 37.4.$$

4. Find the mass of a litre of moist air at 18°C ., the dew-point being 5°C ., and the barometric height 757.5 mm .

5. The relative humidity of air at 16°C ., expressed as a percentage, is 22.5 ; find the dew-point.

6. Two cubic metres of air, at 14°C ., are found to contain 18.56 grams of moisture. Find the dew-point and relative humidity of the air.

7. Twenty litres of moist air, at 15°C ., are drawn through a chemical hygrometer, and found to contain 0.1863 grams of moisture. What is the hygrometric state of the air?

CHAPTER X.

TRANSMISSION OF HEAT.

81. HEAT is transmitted from one point to another in three different ways.

- I. Conduction.
- II. Convection.
- III. Radiation.

In conduction heat is transmitted through the mass of a substance in the direction of decrease of temperature, and is effected by the transfer of heat from each particle to adjacent particles at a slightly lower temperature. A familiar example of conduction is found in the passage of heat along a poker which has one end placed in the fire; it is well known that the heat passes up the poker from the end which is in the fire, that is, it is transmitted in the direction of decrease of temperature.

In convection heat is transmitted through the substance of a body, by means of the motion of heated particles from one point of the body to another. This mode of transmission of heat can, therefore, take place only in liquids and gases. The heating of a liquid by means of a source of heat placed below the vessel containing it (Fig. 51) is a case where convection has most to do with the heating of the entire mass. The portions first heated expand and, thus becoming lighter, rise vertically; to replace this liquid the colder liquid round the sides of the vessel descend, and thus an upward current is formed directly over the flame, and downward currents are set up all round

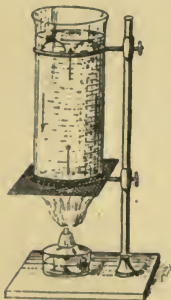


Fig. 51.

the circumference of the vessel. In this way the liquid circulates round and round, and every portion of it is in turn brought under the direct influence of the source of heat. Currents thus set up in a fluid are commonly called *convection currents*; their existence and direction may be demonstrated, in the case mentioned above, by putting a little cochineal or fine sawdust into the liquid.

It must be remembered that although convection currents greatly facilitate the heating of a mass of fluid, by carrying the heated portions away from the source of heat and bringing the colder portions near to it, and also by mixing together the hot and cold portions, yet the transfer of the heat from the source of heat to the fluid, and from the hot to the cold portions of the fluid, takes place by *conduction*, and thus convection may be considered as a hydrostatic phenomenon which results from the expansion of fluids under the action of heat, and accelerates the uniform distribution of heat through a mass of fluid, by setting up a condition of things extremely favourable to the transfer of heat by conduction.

Radiation is entirely different in its nature from conduction and convection. Heat is said to pass by radiation when it is transmitted, from one body to another, through a vacuum, or through an intervening material medium without affecting the temperature of the medium. The heat of the sun reaches us by radiation, as does also the warmth we experience when we stand before a fire. If a thermometer be suspended in a vessel exhausted of air, it can be shown to be susceptible to the influence of a hot body placed in contact with the walls of the vessel; this shows that radiation takes place from the walls of the vessel to the thermometer through the vacuum.

Again, if we hold a piece of metal in front of a fire, or exposed to the direct rays of the sun, it rapidly absorbs heat and its temperature rises, although that of the surrounding air remains constant; this shows that radiation takes place through the air without sensibly affecting its temperature. Bodies which, like air, allow radiation to

take place through them, without themselves being heated, are called *diathermanous*.*

CONDUCTION.

82. Thermal conductivity. Conduction deals, as we have seen, with the transmission of heat through a substance, from particle to particle, in the direction of decrease of temperature; that is, heat always passes from one particle to adjacent particles at a slightly lower temperature. The facility with which this transmission of heat takes place through different substances varies widely with the nature of the substance; this is generally expressed by saying that the **thermal conductivity** of different substances varies with the nature of the substance.

In general, metals are good conductors of heat; organic substances, such as wood, bone, wool, felt, etc., are bad conductors, brick is also a feeble conductor of heat; stone, marble, slate, glass, etc., are better conductors than organic substances or brick, but their conductivity is much below that of the metals (see Table on p. 152). Several familiar phenomena are explained by this difference in the conductivities of different substances; for example, in the hot room of a Turkish bath, where all the objects are at the same temperature, metallic objects feel much hotter than others and may inflict a burn; similarly, in a cold room metallic objects feel very cold to the touch, whilst wooden objects, carpets, clothing, etc., hardly feel cold at all, and yet all the objects in the room may be shown to have the same temperature by bringing them successively in contact with a thermometer.†

The reason of this difference to the touch is found in the

* It should be understood that no substance is perfectly *diathermanous*, but some substances absorb so little of the radiation transmitted through them that they are considered diathermanous. The term *diathermanous* is strictly comparable with *transparent*, as applied with reference to the transmission of light; no substance is perfectly transparent, but gases, glass, etc., are very nearly so.

† If the thermometer be initially at the temperature of the hand it will take the temperature of the metallic bodies much more rapidly than that of the other substances in the room.

fact that the metallic objects, being good conductors, rapidly convey heat to or from the hand, and thus a sensation of heat or cold is experienced. The order of the conductivities of different substances may on this account be roughly determined by touching them, when they are at a common temperature, higher or lower than that of the hand, and classifying them according to the intensity of the sensations of heat or cold experienced in each case. The warmth due to the clothing we wear, bedclothes, etc., results from the low conducting power of the material used for such purposes; the heat of the body is allowed to escape only very slowly, and thus the feeling of warmth is maintained.

A good example of this retention of heat by a hot body surrounded by some bad conductor of heat is found in what is called a Norwegian cooking box, which consists of a wooden box having a thick lining of felt inside, so arranged as to leave a central cylindrical space, into which the vessel containing the food exactly fits. The food is partly cooked over the fire in this vessel, and, while still hot, is placed in the box, and the lid, also felt lined, closed over it. The heat is so well retained that the cooking is completed in the box, and after an interval of some hours, the temperature will have fallen through only a few degrees. Bad conductors are as effective in keeping a body cool as in keeping it warm. Thus a piece of woollen material will keep a vessel of hot water warm if wrapped round it; it will also keep a piece of ice from melting; in the one case heat is prevented from passing out from the water, and, in the other, from passing in to the ice.

Liquids and gases are, in general, very bad conductors of heat, the gases being of very low conductivity. For this reason substances such as cotton wool, which, on account of their structure, contain a great deal of air, are very bad conductors of heat. The difference in the conductivity of different substances may be illustrated by a few simple experiments.

If a short rod of wood have one end fitted into a brass tube of the same external diameter, and a piece of thin

paper be wrapped round the junction of the wood and brass, it will be found, on holding the paper, thus wrapped round the rod, for a short time in the flame of a spirit lamp or Bunsen burner, that the portion in contact with the wood is scorched, while that in contact with the brass is not. This is due to the fact that the brass, being a good conductor of heat, conducts the heat away from the paper sufficiently rapidly to prevent its being scorched, whereas the wood, being a bad conductor of heat, is unable to do this. This experiment may be varied by using a number of cylinders of different substances in the place of the wooden one and comparing the effects produced. If a piece of cardboard be used, instead of paper, it will be found that it is scorched wherever the flame comes in contact with it, the reason being that the cardboard, owing to its low conductivity, does not allow the heat to be transmitted through its thickness to the brass and wood beneath, and hence their presence has no influence on the result.

Similarly, if a copper ball be wrapped up in a pocket-handkerchief and held in a flame, the handkerchief will not be scorched, whereas, if the ball be of wood, wool, or other substance of low conductivity, the handkerchief is immediately scorched.

The difference in the thermal conductivity of different metals may be readily shown by taking strips of silver, copper, and iron, and dipping one end of each in a vessel of boiling water. The other end of the silver strip soon becomes very hot; the copper also gets hot, but not to the same degree as the silver, while the iron strip becomes heated to a much less extent than either the silver or the copper.

83. The stationary and variable states. Imagine a bar placed with one end in the fire, and let us consider what takes place as the heat is transmitted outwards from the fire, along its length. The portion in the fire rapidly increases in temperature and finally takes the temperature of the fire; meanwhile, as the temperature of this portion rises the transverse layer adjacent to it receives heat by conduction, absorbs part of this heat to increase its own

temperature, loses a small portion by radiation from its surface and passes on the rest to the adjacent layer where the process is repeated. When this has gone on for some time a state will ultimately be reached when each layer attains a stationary temperature and ceases to absorb any of the heat passed on to it by the adjacent layer nearer the fire. This state has been called the **stationary state**, and the passage of heat along the bar, when this state is attained, depends, for given conditions, on the conductivity of the material of the bar. The stage previous to the attainment of this stationary or permanent state is known as the **variable state**, for, while it lasts, each layer absorbs some portion of the heat it receives, and is consequently rising in temperature.

84. Definition of conductivity. Conductivity has already been defined generally, as that property of a substance which determines the facility with which heat passes through it by direct transmission from particle to particle. In order to give a complete and accurate definition it is, however, necessary to state how conductivity is measured, and to do this we have to consider the conduction of heat, during the stationary state across a portion of a very large plate of metal or any other substance,* when one side is kept at a constant temperature T_1 , and the other at a constant lower temperature T_2 . It can be shown that, under these circumstances, the flow of heat (H) across the plate varies—

1. *Directly* with the area (A) of the portion across which the flow is considered to take place; that is, $H \propto A$. For example, the flow across 2 square metres is double that across 1 square metre.

2. *Directly* with the difference of temperature between the two faces of the plate; that is, $H \propto (T_1 - T_2)$ or $H \propto \theta$. For example, the flow for 100° difference of temperature is 5 times that for 20° .

*The area of the portion considered should be small compared with the whole area of the plate, and every part of it should be sufficiently distant from the edges of the plate to be uninfluenced by the loss of heat from these edges.

3. *Directly* with the time (t) during which conduction goes on; that is, $H \propto t$. For example, the flow during 1 minute is 60 times that during 1 second.

4. *Inversely* with the thickness (x) of the plate; that is, $H \propto \frac{1}{x}$. For example, the flow across a plate 2 cm. thick is half that across a plate 1 cm. thick.

The above results are concisely expressed by writing—

$$H \propto \frac{A \theta t}{x}.$$

Or—

$$H = k \frac{A \theta t}{x} \quad (1)$$

where k is a constant, and the value of the constant for any substance is taken as a measure of the conductivity of that substance. If we consider the flow of heat, in unit time, across unit area of a plate of unit thickness, having unit difference of temperature between its opposite faces, we have—

$$H = k \frac{1 \times 1 \times 1}{1} = k;$$

and thus we may define the *conductivity of any substance as the quantity of heat flowing, in unit time, through unit area of a plate of that substance, of unit thickness and having unit difference of temperature between its faces*. It is of no importance what units are chosen to express the quantities involved in the above definition, so long as they are *definitely stated and consistent*.

Thus, x may be expressed in *centimetres*,

A	“	“	square <i>centimetres</i> ,
θ	“	“	degrees <i>Centigrade</i> ,
t	“	“	seconds,
and H	“	“	<i>Centigrade</i> gram-degrees.

Or—

x	“	“	<i>feet</i> ,
A	“	“	square <i>feet</i> ,
θ	“	“	degrees <i>Fahrenheit</i> ,
t	“	“	minutes,
H	“	“	<i>Fahrenheit</i> lb. degrees.

It will be seen that the units for A depend on those chosen for x , and the units for H on those of θ .

The most convenient system of units to employ is that known as the C. G. S. system, which involves the centimetre gram, second, and degree Centigrade.

From equation (1) above we have—

$$k = \frac{H x}{A \theta t}. \quad (2)$$

This relation gives us the *mean conductivity* of the substance between the temperatures T_1 and T_2 .

Conductivity as defined in this article is frequently called **absolute conductivity**.

86. Experiments illustrative of conductivity. The following experiments illustrate, in a general way, the questions considered above.

1. Two bars of different metal, but of the same size and shape, and having an exactly similar surface, are placed, end to end, as shown in Fig. 52. To the under surfaces



Fig. 52.

small wooden balls are attached, by wax, at equal distances along the length of the bars, which are then heated at their contiguous ends. As the heat is transmitted along their length, the wax melts and the balls drop off in succession. Now the first ball will drop off directly the temperature of the point at which it is attached becomes equal to the melting point of wax, and, provided the distance of this point from the source of heat be not too great, the time required for this to take place depends not only on the conductivity, but also on the specific heat of the material of the bar; because, the lower the specific heat of this

material, the greater will be the rise of temperature produced in a given mass of it by the heat supplied in a given time. For this reason the balls may begin to drop off first from the bar which has the lower specific heat, but the greatest number of balls will ultimately drop off from the bar which has the greater conductivity; because, the greater the flow of heat along the bar, the further will the rise of temperature necessary to melt the wax be transmitted. †

2. If we take two small cylinders, of equal length, one of iron and the other of bismuth, and, after having coated one end of each with wax, place the other end on a hot copper plate, it will be found, that if the cylinder be not too long, and the temperature of the plate be sufficiently high, the wax on the upper end of the bismuth melts first, although the conductivity of iron is more than six times that of bismuth.

This is explained by the fact that the thermal capacity of the bismuth is much less than that of the iron, and consequently it requires more heat to raise the temperature of the iron, through a given range of temperature, than it does to raise the bismuth through the same range; hence, although the heat transmitted along the bismuth is less than that along the iron, the rise of temperature produced in the former is greater. When, however, the permanent state is attained a greater quantity of heat will be transmitted through the iron than through the bismuth, and the temperature of the upper end of the iron cylinder will ultimately be the greater.

3. Ingenhousz's apparatus (Fig. 53). This consists of

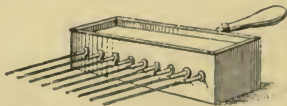


Fig. 53.

a box into which rods of the same thickness, but of different

metals, are fixed, as shown in the figure. The ends of the rods pass into the box, and are raised to a high constant temperature by filling the box with hot water or oil. The portions outside the box are coated with wax, so that, as heat is transmitted along their length, the wax gradually melts. When the permanent state is attained the relative lengths along which melting takes place depend on the conductivities of the metals employed, and it can be shown that the conductivities are in the ratio of the *squares* of the lengths along which the wax has been melted. Thus, if the length to which melting has proceeded on A is twice that on B, then the conductivity of A is four times that of B.

88. Experimental determination of absolute conductivity. Theoretically it would appear that the simplest method of determining absolute conductivity of a substance would be to determine the quantity of heat transmitted through a plate of the substance, of given area and thickness, and having a known difference of temperature between its faces, and then to apply the formula of Art. 84.

This method is impracticable for two reasons.

(1) It is difficult, if not impossible, to maintain the two faces at constant *known* temperatures.

(2) It is impossible to avoid *lateral* loss of heat by radiation and convection from the edges of the plate.

On this account, the most usual method of investigating conductivity is to take a bar of the substance, the conductivity of which is to be determined, and, keeping one end at a high constant temperature, to note the temperature assumed at different points in its length when the permanent state is attained. From the temperature data thus obtained, combined with an experimental determination of the loss of heat by radiation and convection from the surface of the bar, the absolute conductivity of the substance can be calculated.*

* See "Text Book of Heat," Arts. 87, 88.

89. Relative conductivity. The method described in the preceding article may be, and has been, employed for determining the relative conductivities of different metals. The method of reducing the observations is beyond the scope of this work. Rough determinations of relative conductivity may be made by the method illustrated by Ingenhousz's apparatus. In all such methods, however, two things must be carefully attended to.

(1) The observations must be made during the permanent state. [Cp. Art. 86, (1) and (2).]

(2) The surface of the bars must be of the same extent and nature.

This last condition is of equal importance with the first, for the fall of temperature along a bar evidently depends on the rate of loss of heat from its surface; and this depends on the nature and extent of that surface. Wiedemann and Franz, in their experiments on relative conductivity, by the method of the preceding article, employed thin rods of the same size and shape, and electro-plated their surfaces to ensure exact similarity in their nature.

TABLE OF CONDUCTIVITIES.

Substances.	Relative Conductivity.	Absolute Conductivity k <i>C.G.S. units.</i>	Diffusivity κ <i>C.G.S. units.</i>
Silver . . .	100.0	1.527	2.580
Copper . . .	73.6	1.000	1.183
Gold . . .	53.2	.723	1.156
Brass . . .	23.6	.321	.407
Zinc . . .	19.0	.258	.382
Tin . . .	14.5	.197	.482
Iron . . .	11.9	.164	.187
Steel . . .	11.6	.158	.181
Lead . . .	8.5	.115	.328
Platinum . . .	8.4	.114	.132
German silver . . .	8.0	.109	.128
Palladium . . .	6.3	.086	.126
Bismuth . . .	1.8	.024	.079
Ice38	.0052	.0114
Roll Sulphur04	.0006	.0014
Coal02	.0003	.0012

90. Conductivity of liquids. The determination of the conductivity of fluids is very difficult for three reasons.

1. Convection currents have to be avoided.

2. From their nature, fluids have to be enclosed in vessels, and the conduction by the walls of the vessel interferes with the result.

3. Their conductivity is very low.

All liquids, except mercury and other liquid metals, have very low conductivities. The transmission of heat through the substance of a liquid is mainly due to convection, and if the circumstances of heating are such as to prevent the formation of convection currents, heat is transmitted only very slowly through the liquid; thus, if a piece of ice be placed in a test tube and water poured on so as to nearly fill the tube, the water in the upper part of the tube may be boiled without affecting the ice at the bottom, thus showing that little, if any, heat is conducted down through the water in the tube.

92. The Davy lamp. When we apply the flame of a lighted match to gas issuing from a jet, the temperature of the gas is raised above its *temperature of ignition*, and combustion commences, and continues so long as a supply of oxygen is obtainable and the temperature remains above the ignition temperature of the gas.

If a piece of good conducting material be placed in a flame, it may conduct the heat away so rapidly, that the temperature falls below the temperature of ignition and combustion ceases. This effect may be shown by holding a coil of copper wire in a small gas flame; the wire rapidly conducts away the heat, and the flame is soon extinguished. The action of wire gauze on a flame is explained in the same way. If the gauze is lowered on to the flame it seems to crush it down; but it is, in reality, gradually extinguishing it. The process of combustion ceases wherever the gauze is in contact with the flame, because the former conducts the heat away sufficiently rapidly to lower the temperature below that at which combustion takes place. Similarly, if the gas be lighted above the gauze, the flame

appears only above the gauze, for the fall of temperature produced by the latter in its immediate neighbourhood, prevents the flame from passing through. This principle has been utilised in the Davy lamp; the flame is surrounded by wire gauze, so that, when placed in a combustible or explosive atmosphere, the gases penetrating into the flame, burn inside the gauze, and thus indicate the dangerous nature of the atmosphere; but, as the flame cannot pass through the gauze, the explosion that would be produced by a naked light is averted. It has been shown, in recent years, that the Davy lamp is not in all cases an efficient protection against explosions in mines.

CONVECTION.

93. We have already explained the nature of this mode of transmission of heat, and shall now briefly consider some illustrations of its action.

If a gas flame, or other heated body, be placed in the middle of a room containing air at a uniform temperature, convection currents are at once set up; the air in the neighbourhood of the flame ascends, and is replaced by that supplied by downward currents near the walls of the room. The existence of these currents is readily shown by holding a piece of smoking paper in different parts of the room, which should, however, be free from external draughts. The direction taken by the smoke indicates the direction of the convection currents.

All systems of ventilation are merely methods for establishing convection currents between the outside air and the air in the room to be ventilated, in such a way as to promote free circulation of the air, without creating draughts. The draught in a chimney is also an instance of convection; the heated air ascends, and is replaced by colder air from the room, and thus a fire in a room to which the outer air has some means of access is an effective means of ventilation.

The familiar phenomena of winds are good examples of convection currents, set up in the atmosphere by unequal heating. As examples we may consider :

1. **The trade winds.** The surface of the earth within the tropics becomes greatly heated by the sun, and this heat is communicated to the air in contact with the earth, causing it to ascend. To replace this air, a current of colder air sets in from the polar and temperate regions, thus tending to produce a north wind in the northern hemisphere and a south wind in the southern hemisphere. To determine the actual direction of the wind it is, however, necessary to consider the rotation of the earth from west to east. A point on the surface of the earth near the equator has a velocity, from west to east, of more than a thousand miles per hour; but this velocity decreases as we approach the poles, where it becomes zero. Hence, if a mass of air start from a place having a velocity, from west to east, of eight hundred miles per hour, and flow towards the equator, its velocity relative to the surface of the earth will gradually tend *towards the west*, until on reaching the equator it will have a **relative** velocity of more than two hundred miles per hour *from east to west*.

Hence, if, at any point on the borders of the temperate and tropical zones in the northern hemisphere, N S (Fig. 57) represents the velocity of the current of air towards the equator, and N W represents its relative velocity towards the west, then N A represents in magnitude and direction the actual velocity of the wind relative to the surface of the earth. Similarly S' A' represents the magnitude and direction of the wind in the southern hemisphere. Thus, a north-east wind* is produced in the northern hemisphere, and a south-east wind in the southern hemisphere. These winds are known as the **trade winds**. Corresponding to these lower currents there are upper ones, known as the **return trades**, flowing in the northern hemisphere from the south-west, and in the southern hemisphere from the north-west. The existence



Fig. 57.

* A north-east wind is a wind *from* the north-east.

and direction of these upper currents has been demonstrated by the direction in which volcanic dust has been carried from the tropics.

2. **Land and sea breezes.** Land absorbs radiant heat more rapidly than water, and, owing to its smaller specific heat, is raised to a higher temperature by the heat thus absorbed. For the same reason land loses heat more rapidly than water. Moreover, evaporation takes place from the surface of the water, and this tends to keep it cool (Art. 67). Hence, during the day the land gets heated by solar radiation to a greater degree than the sea, and the air in contact with it also gets heated, expands, and, rising, is replaced by the cooler air over the sea, thus causing a **sea breeze**. After sunset, however, the land cools more rapidly than the sea, and ultimately the air over the land becomes colder than that over the sea; the direction of the current of air is then reversed, and during the night a **land breeze** prevails.

A large number of ocean currents are surface currents, due to the action of prevailing winds; but some are due to convection. Thus the cold in the Arctic regions causes water to sink and flow towards the equator as a deep current, while the surface water at the equator flows towards the poles, to replace the water carried away by this under current.

Convection in fluids counterbalances, in a great measure, their low conductivity. Thus, water may be boiled in a paper vessel, because the heat is carried away from the paper, by convection, sufficiently rapidly to prevent the latter being scorched (Art. 82). Similarly, in a mass of gas, uniformity of temperature is established by convection more rapidly than in a mass of good conducting material by conduction (Art. 43, 6).

Another application of convection currents is found in the system of heating buildings by hot-water pipes. The water in the boiler, which is placed at the lowest available position, is heated directly by the fire and rises through the outflow pipe, which should emerge from the boiler at its highest point, and, circulating round the network of

pipes, return by the return pipe which enters the boiler at its lowest point. The rapidity of the circulation depends on the difference between the average densities of the water in the outflow pipe, from the boiler to the highest point reached, and that in the return pipes from the highest point to the boiler. This difference of density itself depends on the difference of temperature, which should therefore be as great as possible, and for this reason the outflow pipe should pass vertically upwards as far as possible, so as to have a long vertical column of water at the highest possible temperature.

The transmission of heat by this system of hot-water pipes from the boiler furnace to a person feeling the warmth is a good example of the different methods of transmission of heat. The heat passes from the furnace to the water in the boiler by *conduction* through the boiler plates. It is transmitted through the mass of the water by *convection*, passes through the pipes to the air in the room by *conduction*, and this air, heated by *convection*, in its turn warms any person whom it surrounds. Heat is also *radiated* from the pipes to the different persons and objects in the room.

CALCULATIONS.

95. IN connection with the subject matter of the preceding pages, it will be useful to summarise the following points involving quantitative relations :—

1. **Absolute conductivity.** In Art. 84, we have the important relation expressed by—

$$H = k \frac{A\theta t}{x}. \quad (1)$$

Also, deduced from this, we have—

$$k = \frac{Hx}{A\theta t}. \quad (2)$$

[Of these (1) only should be learnt; from it, (2) can be obtained when required.]

EXAMPLES XI.

Reference should be made where necessary to the table of conductivities on page 152.

1. Find the quantity of heat that will be transmitted, in 1 hour, across a plate of copper 1 sq. metre in area and 5 cm. thick, the difference between the temperatures of its faces being 10° C.

From (1) above we have—

$$H = k \frac{A\theta t}{x}.$$

Adopting the C. G. S. system of units, we have—

$$\begin{aligned} k &= 1 \text{ (see table, p. 152),} \\ A &= 1 \text{ sq. metre} = 10000 \text{ sq. cm.,} \\ \theta &= 10^\circ \text{C.,} \\ t &= 1 \text{ hour} = 3600 \text{ seconds,} \\ x &= 5 \text{ cm.} \end{aligned}$$

$$\begin{aligned} \therefore H &= \frac{10000 \times 10 \times 3600}{5} \\ &= 72000000 \text{ gram-degrees.} \end{aligned}$$

2. It is found that 9162000 gram-degrees of heat are transmitted, per minute, across a sheet of silver, 100 sq. cm. in area and 1 mm. thick, with a difference between the temperatures of its faces of 100° C. Find, in C. G. S. units, the absolute conductivity of silver.

From (1) above we have—

$$H = k \frac{A\theta t}{x} ;$$

and therefore—

$$k = \frac{Hx}{A\theta t}.$$

Here—

$$\begin{aligned} H &= 9162000 \text{ gram-degrees,} \\ x &= 0.1 \text{ cm.,} \\ A &= 100 \text{ sq. cm.,} \\ \theta &= 100^\circ \text{ C.,} \\ t &= 60 \text{ seconds.} \end{aligned}$$

Substituting, we get—

$$k = \frac{9162000 \times 0.1}{100 \times 100 \times 60} = 1.527.$$

5. Peclet has stated that the quantity of heat which passes, in an hour, through a plate of lead 1 sq. metre in area and 1 cm. thick, with a difference of 1°C. between the temperature of its surfaces, is 1383 kilogram-degrees. What value does this give for the absolute conductivity of lead in the C. G. S. system?

6. The absolute conductivity of copper in the C. G. S. units is 1; how many heat-units will pass, per minute, across a plate of copper, 1 metre long, 1 metre broad, and 5 cm. thick, when its opposite faces are kept at temperatures differing by 100°C. ?

7. The thermal conductivity of felt, in C. G. S. units, is 0.000087; find the quantity of heat that is transmitted, in one hour, through a layer of felt 1 cm. in thickness and 20 sq. cm. in area, when its opposite faces are kept at temperatures differing by 20°C.

9. A square metre of a substance, 1 cm. thick, has one side kept at 100°C. , and the other, by means of ice, at 0°C. In the course of 10 minutes one kilogram of ice is melted by this operation. Calculate the conductivity of the substance, assuming the latent heat of water to be 80.

EXAMINATION QUESTIONS.

QUESTIONS SET AT LONDON UNIVERSITY EXAMINATIONS.

Matriculation.

1. A source of heat is applied equally to the extremities of two similar bars, one of copper and one of iron; and on each bar there is a piece of phosphorus two inches from the source of heat. That on the copper takes fire first. Does this experiment entitle us to conclude that copper has greater thermal conductivity than iron? If not, explain in what respects it is deficient as a proof. *Jan., 1877.*

2. How would you compare the thermal conductivities of brass and copper? Two equal cylinders, one of iron and the other of bismuth, are covered with wax and simultaneously placed on end on a hot metal plate. At first the melting of the wax advances more rapidly on the bismuth bar; but after it has melted about an inch up both cylinders, the melting advances the more rapidly on the iron bar. Account for these phenomena. *June, 1880.*

3. Define latent heat, specific heat, capacity for heat, coefficient of cubic expansion, and thermal conductivity. How would you determine the capacity for heat of a copper vessel? *Jan., 1881.*

4. Define the dew-point.

A cylinder which we may suppose impervious to heat is closed by a piston, and contains steam, with a little water, at 100°C . The piston is suddenly depressed so as to compress the steam. State fully what happens. *Jan., 1882.*

5. Describe an experiment which shows that water is a very bad conductor of heat. *June, 1882.*

6. What are the laws of pressure in a mixture of gases and vapours? Explain the principle of, and describe the method of using, the wet and dry-bulb hygrometer. *Jan., 1883.*

7. Describe and explain the method of using some form of dew-point hygrometer, and show how to determine the humidity of the air by means of it. *June, 1883.*

8. Explain the expression "tension of aqueous vapour." How is the pressure of the aqueous vapour in the atmosphere connected with the dew-point? Describe some method of determining the dew-point. *Jan., 1884.*

9. A building is heated by hot-water pipes. How does the heat get from the furnace of the boiler to a person in the building? What would be the effects on the temperature of the more distant parts of the building of coating the pipes near the boiler (*a*) with woollen felt, (*b*) with dull black lead? *June, 1887.*

LIGHT.

CHAPTER I.

INTRODUCTORY.

1. Light is the external physical cause of the sensation called sight.

There is strong evidence in favour of supposing this external cause to consist of vibratory motion of the ether; but, whatever may be the nature of light, there are some fundamental properties, established by experiment, which may be studied quite independently of any hypothesis on this point.

In the pages that follow we propose to study in this way a few of the more important of these fundamental properties; but, as the undulatory theory is now so completely established, reference will be made to it whenever it seems advisable.

2. Preliminary definitions. Bodies which are of themselves capable of exciting the sensation of sight are termed *self-luminous*; bodies which, in themselves, are not luminous become so in the presence of a self-luminous body.

Any space through which light can pass, whether it be occupied by matter or not, is termed a *medium*.

A *homogeneous medium* is one that is uniform throughout in structure and properties. Media are described with reference to the facility with which light passes through them, as being more or less transparent. A *transparent medium* transmits the light incident upon it; but when light travels through any medium a certain proportion of it,

depending in amount on the nature and thickness of the medium, is absorbed; hence no medium is perfectly transparent. Bodies which do not transmit light are said to be *opaque*. The terms *transparent* and *opaque* refer to a difference in degree more than in kind; probably all substances transmit light if sufficiently thin layers are considered. For this reason it is perhaps more correct to speak of transparent and opaque *bodies* than to apply these terms to *substances*. We shall often find it necessary to consider the light travelling along a particular line. For this purpose a *ray* of light is defined as the portion of light enclosed by a hollow cone of infinitely small angle, whose axis is the particular line considered.

A *pencil* of light is a collection of adjacent rays, and may be *divergent*, *convergent*, or *parallel* —

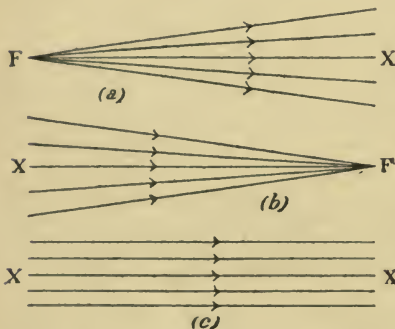


Fig. 1.

that is, the component rays may diverge from, or converge to a point, or run parallel (Fig. 1, *a*, *b*, *c*). A convergent or divergent pencil has the form of a cone of small, but finite angle, while a parallel pencil is enclosed by a cylinder of small cross section.

The *axis* of a pencil, *FX* (Fig. 1), is the central ray passing along the geometrical axis of the figure of the pencil, and the point, *F*, from or to which the rays of a pencil diverge or converge is called its *focus*. The focus of a parallel pencil is at infinity. Light made up of divergent, convergent, or parallel pencils is said to be divergent, convergent, or parallel; when light comes from a very distant source—for instance, from the sun, moon, stars, etc.—it is considered to be parallel, although, strictly speaking, it is very slightly divergent.

CHAPTER II.

RECTILINEAR PROPAGATION OF LIGHT.

3. Light travels in straight lines through the same homogeneous medium. Many familiar phenomena point to the fact that light travels through the same homogeneous medium in straight lines. If two screens be each pierced with a small hole and then held one in front of the other, in such a position (Fig. 2) that the two holes and a candle flame are in the same straight line, a ray of light can pass from the candle through the holes to an eye placed in the same straight line behind the screens; but, if either of the screens be but slightly displaced in its own plane, the candle becomes invisible.

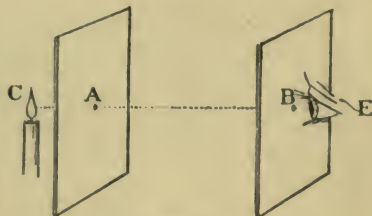


Fig. 2.

Similarly, if a scale be laid on the bottom of a vessel, and looked at over the edge of the vessel, in the way indicated in Fig. 3, it will be found that the line EAS is a straight line. In both these experiments the same medium (air) extends between the eye and the object seen; but if, in the latter example, water be poured into the vessel, it will be found that a point, S' , on the scale can be seen, and that EAS' is not a straight line. Hence, when light passes from one medium to another, it is in general bent out of its direct rectilinear path; and, from what has been said, it is evident that the bending must take place at the surface of separation of

the two media. When, however, a ray of light travels through a non-homogeneous medium, it may suffer gradual and continuous change of direction, if the change in the properties of the medium along its path are also gradual and continuous ; it is only on passing from one homogeneous

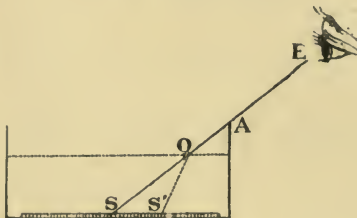


Fig. 3.

medium to another that sudden change of direction takes place. The magnitude and direction of this change depend on conditions which we shall consider more fully in the chapters on Refraction and Dispersion.

It should be noticed in connection with this point that the eye takes no cognizance of change of direction in a ray of light ; every object is seen 'in the direction taken by the axis of the pencil of light which enters the eye. For example, if light, starting from O (Fig. 4), be bent, as indicated in figure, then O appears to the eye to be at O'. The point O' is the *virtual focus* of the pencil entering the eye, called *virtual*, because its rays do not really diverge from O', but appear to do so.



Fig. 4.

4 The pinhole camera. If a sheet of cardboard, pierced, at its centre, with a large pinhole, be placed between a candle and a thin paper screen, shaded from external light,

a more or less distinct representation of the candle flame will be seen, in an inverted position, on the screen (Fig. 5).

If the cardboard form the front, and the screen the back of a closed box, the representation can be seen very distinctly from behind, through the paper (or ground glass) screen. The explanation of this is simple. Let $A B$ (Fig. 6)

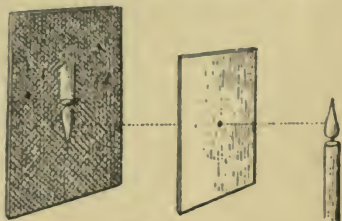


Fig. 5.

represent the candle flame or other brightly illuminated object, O the hole in the cardboard, and $S S$ the screen. From every point on $A B$ rays

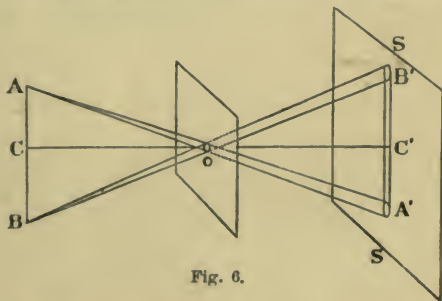


Fig. 6.

are given off in all directions, and consequently from every point of $A B$ a small pencil of rays passes through O , and forms a small circular or elliptical spot on the screen. The result of this is that we have on the screen an assemblage of nearly circular spots, which, owing to the crossing of the rays at O , define an *inverted* representation of the object $A B$. If these spots are large they overlap one another, and the representation is blurred and indistinct; hence, in order to obtain a well-defined picture of $A B$ on the screen, the aperture at O must be very small, for, it is evident, that the size of the spot on the screen depends, for given positions of $A B$ and $S S$, upon the size of aperture.

5. Shadows. The formation of shadows is a direct consequence of the rectilinear propagation of light. If an opaque body (B) be placed so as to intercept a portion of the light emitted by a luminous point (L), the cone of light incident on the surface of the body is stopped, and the space beyond, enclosed by the geometrical continuation of this cone, is screened from the rays diverging from L.

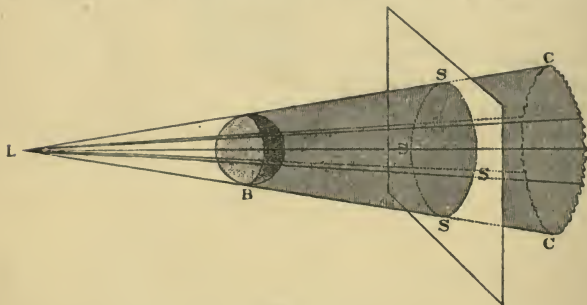
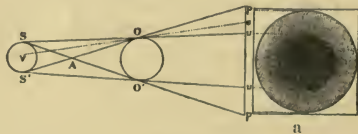


Fig. 7.

The cone here considered (LCC') is called the *shadow cone*, and its trace ($SSSS$) on any surface intersecting it beyond B, outlines the *shadow* cast on this surface. When, however, the source of light is not a luminous point, but a luminous body, the case is somewhat more complicated. Let SS' (Fig. 8, *a*) represent a spherical source of light, and OO' an

Fig. 8, *a*.

opaque sphere placed near it. Consider the cone $SS'uu$ which touches SS' and OO' externally; it is evident from the figure that *no* light from SS' falls within the portion of this cone lying beyond OO' ; and for this reason it has been called the cone of *total shadow*, or the cone of the *umbra*;

and the portion of it, just referred to as being completely screened from the light, is called the umbra. Consider again the double cone $SS'A pp$, touching SS' and OO' internally, and having its apex at A . This is the cone of *partial shadow*, or the cone of the *penumbra*, and the portion of it beyond OO' , and surrounding the umbra, is known as the *penumbra*. From any point in the cone, not within the total shadow, a portion of the source of light can be seen, and for this reason the shadow is only partial. The depth of shadow at any point depends on the extent of the source invisible from that point; to an eye placed at e all below ev is invisible, while all above is visible; hence, at points near the outer boundary of the penumbral cone, the shadow is very light, but gradually deepens as we approach the outer boundary of the umbral cone.

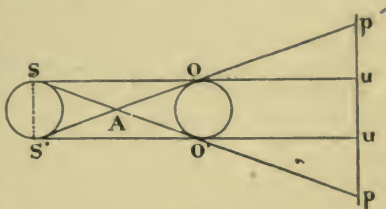


Fig. 8, b.

The penumbral cone always has the form of a cone diverging from a point (A) lying between SS' and OO' , but the form of the umbral cone depends on the relative size of the source of light and the opaque body; when the latter is the greater, the cone diverges from a point behind the former (Fig. 8, *a*), when equal it takes the form of a cylinder (Fig. 8, *b*), and when smaller the cone converges to a point beyond the opaque body (Fig. 8, *c*). If the shadow of the opaque body be cast upon a suitably placed screen (Fig. 8, *a'*),

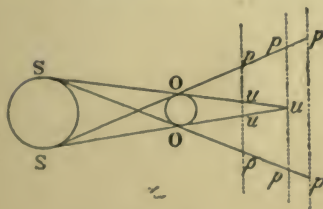


Fig. 8, c.

it will be found to consist of a central region of total shadow, the umbra, surrounded by a zone of partial shadow,

the penumbra. The former is of uniform depth all over, but the latter passes gradually from the total shadow of the umbra to a complete absence of shadow at its outer boundary, and consequently neither its outer nor its inner edge is sharply defined. The relative size of the umbra and penumbra in any particular case depends upon the conditions illustrated in Fig. 8, and upon the position of the screen. The reader will find it instructive to draw diagrams for a number of different cases.

CHAPTER III.

PHOTOMETRY.

6. **Light as a measurable quantity.** Light, like radiant heat, is undoubtedly a form of energy, and, as such, is capable of measurement. The quantity of light in any space at any instant is measured by the corresponding amount of energy in that space at the instant considered, and the physical intensity of the light is measured by the energy transmitted through that space in unit time. This measurement cannot be made directly; but, for light of a given colour, the intensity is proportional to the brightness as perceived by the eye, and also to its heating effect. There are thus two methods of measuring the intensity of light—the *photometric* and the *calorimetric*.

In this chapter we shall consider the photometric method only.

8. **Intensity of illumination of a surface at a point.** Let q denote the quantity of light incident on a small element of a surface of area s , then the limit of the ratio $\frac{q}{s}$, when s , and therefore q , are indefinitely diminished gives the intensity of illumination of the surface at the point at which s vanishes. If $\frac{q}{s}$ is constant for all points, then the surface is said to be *uniformly illuminated*; and if Q denote the quantity of light incident upon a portion of the surface of area S , then $\frac{Q}{S}$ determines the intensity of this uniform illumination.

9. Law of inverse squares. Let P (Fig. 9) represent a luminous point, and let us conceive this point to be surrounded by a sphere $S_1 S_1 S_1$ of radius, R_1 , having its centre at P. Then the inner surface of this sphere will be uniformly illuminated, and the intensity of illumination, I_1 , is given by—

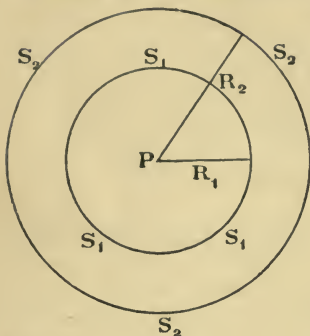


Fig. 9.

$$I_1 = \frac{Q}{4\pi R_1^2}^*$$

where Q denotes the total quantity of light emitted by P. Similarly, if we consider the sphere $S_2 S_2 S_2$, the intensity of illumination of its inner surface is given by—

$$I_2 = \frac{Q}{4\pi R_2^2}.$$

Hence we have—

$$\frac{I_1}{I_2} = \frac{Q}{4\pi R_1^2} \bigg/ \frac{Q}{4\pi R_2^2} = \frac{R_2^2}{R_1^2}.$$

That is, *the intensity of illumination, of any uniformly illuminated surface, is inversely proportional to the square of its distance from the source of light; or, in more general terms, the intensity of illumination, at any point of a surface, is inversely proportional to the square of the distance of that point from the source of light.*†

It follows from this that if I_1 denote the intensity of illumination of a surface at unit distance from the source of light, then the intensity of illumination on a similarly placed surface at a distance, R , is given by—

$$I = \frac{I_1}{R^2} \text{ for, } \frac{I}{I_1} = \frac{1}{R^2}; \text{ that is, } I = \frac{I_1}{R^2}.$$

* The area of the surface of a sphere of radius $R = 4\pi R^2$.

† In this and the following articles the dimensions of the source of light are considered to be so small, compared with the other distances involved, that it may be treated as a luminous point.

11. The illuminating power of any source of light. The intensity of any source of light is proportional to its illuminating power. *This quantity is measured by the intensity of illumination of unit area of a surface placed at unit distance from the given source, the light being incident normally on this surface.* If I_1 denote the illuminating power of a given source of light (A), then the intensity of illumination produced by this source on a surface, at a distance R_1 , is given by—

$$I = \frac{I_1}{R_1^2}. \quad (\text{Art. 9.})$$

Similarly, if another source of light (B) be so placed as to produce the *same* intensity of illumination (I), then—

$$I = \frac{I_2}{R_2^2}$$

where I_2 denotes the illuminating power of B, and R_2 its distance from the illuminated surface. Now, equating these two expressions for I, we have—

$$\frac{I_1}{R_1^2} = \frac{I_2}{R_2^2}.$$

That is—

$$\frac{I_1}{I_2} = \frac{R_1^2}{R_2^2}.$$

This shows that *the illuminating powers of different sources of light are directly* proportional to the squares of the distances they must be placed from a given surface, in order to produce on it the same intensity of illumination.* It should be noticed that the above is true only when the angle of incidence† is the same in each case; hence it is evident, that in an experimental comparison of illuminating powers care must be taken to satisfy this condition.

Note.—It is important to distinguish between the quantities *intensity of light*, *illuminating power*, and *intensity of illumination*. Intensity of light is a quantity involving energy (Arts. 6, 7); illumi-

* This statement must be carefully distinguished from that made at the end of Art. 9.

† The intensity of illumination varies as the cosine of the angle of incidence of the light. See "Text Book of Light," Art. 10.

nating power of a source of light is proportional to the intensity of the light, and is measured as stated in Art. 11; intensity of illumination refers to the surface illuminated, and not, like the other two quantities, to the source of light (Arts. 8, 9).

12. Photometers. Photometry is the experimental comparison of the illuminating powers of different sources of light, and the different forms of apparatus by which this comparison is effected are called *photometers*. The practical unit employed in photometry is the light emitted by a standard sperm candle (six to the pound), burning 120 grains per hour; and hence the illuminating power of any source of light is generally expressed as being equivalent to that of a certain number of standard candles. It has been found that the eye is unable to estimate the ratio of the intensity of illumination due to different sources of light, but that it is a correct judge of the equality of the

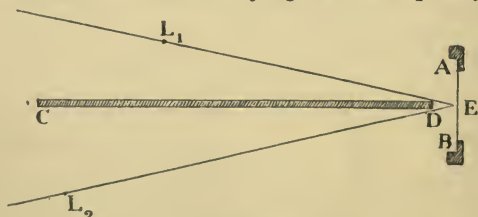


Fig. 11.

illumination of two adjacent surfaces. For this reason all methods of photometry depend on the equalisation of two illuminations, and the details of the construction of photometers are devised to facilitate this adjustment.

Foucault's photometer. This photometer (Fig. 11) consists of semi-transparent screen, A B, of thin paper, ground glass, or thin white porcelain, fixed vertically in front of, and at right angles to a partition, C D, which is movable by means of a screw in the direction of its length. The two sources of light to be compared, L_1 and L_2 , are placed on opposite sides of this partition in such positions that the angle $L_1 E L_2$ is bisected by C D. By this arrangement one portion of the screen is illuminated by one

source and the other portion by the other; and, by adjusting the position of CD until these separately illuminated portions become contiguous, their illuminations may be more accurately compared. When both portions appear equally bright the comparison is complete, and we have, if L_1 and L_2 represent the final positions of the sources of light—

$$\text{Illuminating power of } L_1 = (E L_1)^2 *$$

$$\text{Illuminating power of } L_2 = (E L_2)^2$$

Rumford's photometer. In this photometer the illuminating powers of two sources of light are compared by

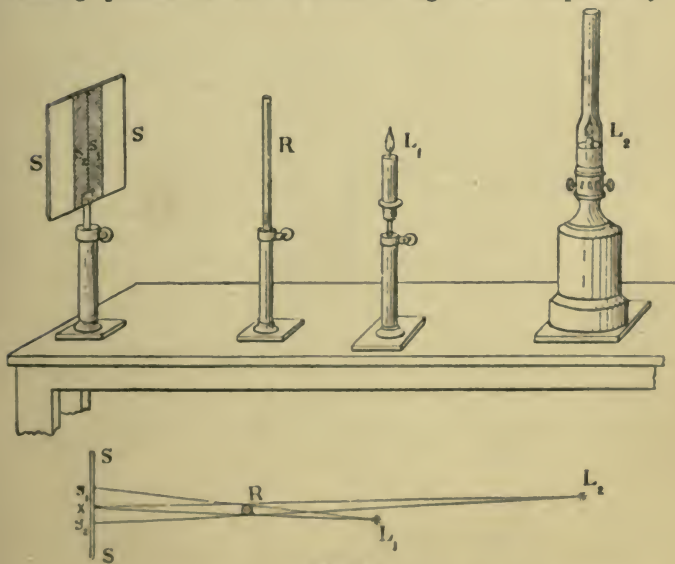


Fig. 12.

adjusting to equality the intensities of the two shadows of a vertical rod cast on a screen by the two given sources. The lights (L_1, L_2), rod (R), and screen (SS) are arranged, as shown in Fig. 12, so that the two shadows ($S_1 S_2$), whose

* The dimensions of the screen are small compared with the distances EL_1 and EL_2 .

edges should be well defined, appear close together, and of equal intensity. In this way, since each shadow is illuminated by the source to which the other is due, equality of intensity of the shadows cast by L_1 and L_2 means equality of illumination due to L_2 and L_1 ; and hence we have, as in previous case—

$$\frac{L_1}{L_2} = \frac{(L_1 X)^2}{(L_2 X)^2}.$$

Bunsen's photometer. Bunsen has devised a very simple form of photometer. If a sheet of paper, having a spot of grease on it, be held up to the light, it will be seen that the spot of grease is semi-transparent, and looks brighter than the rest of the paper when viewed from the side remote from the light, but darker when seen from the other side. The reason of this is evident; more light passes through the region of the grease spot than through the rest of the paper, and hence when seen from the side remote from the light it looks brighter than the rest of the screen through which little or no light passes; when looked at on the other side, however, the spot looks comparatively dark, because a large proportion of the light incident upon it passes through, and is therefore not spent in illuminating its surface. It will now be understood that if a suitable paper screen, having a grease spot at its centre, be placed between two sources of light, and its position adjusted until the spot cannot be seen on either side, except by close inspection, then the screen must be equally illuminated on both sides.

If L_1 , L_2 , and SS (Fig. 13) represent the relative positions

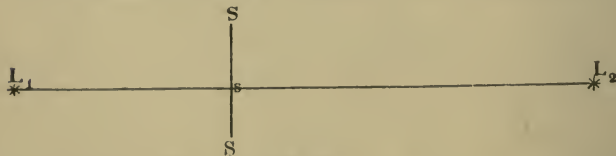


Fig. 13.

of the lights and the screen when finally adjusted, we have—

$$\frac{L_1}{L_2} = \frac{(L_1 s)^2}{(L_2 s)^2}.$$

In practically carrying out the necessary measurements, at least four different adjustments should be made: (1) Adjust for disappearance of the spot when seen from the side of screen facing L_1 . (2) Turn the screen round through 180° , and again adjust for disappearance of spot from the same surface now facing L_2 . (3) Repeat (1) and (2) with the other surface of the screen.

The screen is usually mounted in a light frame, so that it can be easily turned round in its stand, or as a whole.

In all photometric measurements the light to be compared should be of the same colour. If this is not the case it will be found impossible to adjust accurately for equal illumination, owing to the difference in colour of the illuminated surfaces. In general, different sources of light emit differently coloured rays in different proportions (Art. 62), so that an accurate comparison of intensity can only be made by means of an instrument which forms the light from each source into a spectrum (Art. 63), and admits of a comparison of corresponding parts of the spectra so formed.

CALCULATIONS.

13. THE calculations connected with the subject-matter of the preceding chapters are simple applications of the elements of geometry or algebra to the principles there explained, and need no further illustration than is afforded by the worked examples given below.

Note.—In Chapter III. we have made use of the term *cosine*, and in succeeding chapters it will be necessary to make frequent use of the term *sine*. Hence for the convenience of the reader we shall now explain these terms.

Let D A E represent a plane angle. From any point C, in A E, draw C B perpendicular to A D, and cutting A D in B. Now the length of B C, for a given position of C, evidently depends on the magnitude of the angle D A E, but it gives no indication of this magnitude unless the position of C be defined. For this purpose the ratio $\frac{B C}{A C}$ may be considered, and it can be shown geometrically that wherever C be taken on A E this ratio is constant, and is definitely related to the magnitude of the angle B A C. Similarly the ratio $\frac{A B}{A C}$ is constant and bears a fixed relation to the magnitude of B A C. The ratio $\frac{B C}{A C}$ is called the *sine* of B A C, and the ratio $\frac{A B}{A C}$ is called the *cosine* of B A C. In the right-angled triangle B A C, considered with reference to the angle B A C, the side B C is called the *perpendicular*, the side A B is called the *base*, and A C is called the *hypotenuse*. Hence, in general terms—

$$\text{sine } B A C = \frac{\text{perpendicular}}{\text{hypotenuse}} = \sin B A C.$$

$$\text{cosine } B A C = \frac{\text{base}}{\text{hypotenuse}} = \cos B A C.$$

The reader should deduce geometrically the values of these ratios for angles of 30° , 45° , and 60° . These will be found to be—

$$\sin 30^\circ = \frac{1}{2} \quad \cos 30^\circ = \frac{\sqrt{3}}{2}.$$

$$\sin 45^\circ = \frac{1}{\sqrt{2}}. \quad \cos 45^\circ = \frac{1}{\sqrt{2}}.$$

$$\sin 60^\circ = \frac{\sqrt{3}}{2}. \quad \cos 60^\circ = \frac{1}{2}.$$

EXAMPLES I.

1. In a pinhole camera the distance from the aperture in front, to the screen at the back, is 18 inches. Find the relative dimensions of the representation on the screen of an object placed 6 feet in front of the camera.

In Fig. 6, treating the pencils from A and B to A' and B' respectively, as lines we see that the triangles A O B and A' O B' are equiangular, and therefore similar (Euclid vi. 4).

$$\therefore \frac{AB}{A'B'} = \frac{CO}{OC'}.$$

Here, CO = 6 feet and OC' = $1\frac{1}{2}$ feet.

$$\therefore \frac{AB}{A'B'} = \frac{CO}{OC'} = \frac{6}{1\frac{1}{2}} = 4.$$

$$\therefore AB = 4A'B'.$$

2. A circular uniform source of light, 2 inches in diameter, is placed at a distance of 10 feet from a sphere 2 inches in diameter. Calculate, approximately, the diameters of the umbra and penumbra cast on a screen 5 feet beyond the sphere. *Matric., June 1889.*

Here, in Fig. 8 (b)—

SS' = 2 inches; OO' = 2 inches; SO = 10 feet; Ou = 5 feet.
Diameter of umbra = uu = OO = 2 inches.

Diameters of penumbra $\begin{cases} \text{Internal} = uu = 2 \text{ inches} \\ \text{External} = pp = 4 \text{ inches} \end{cases}$; for, from the triangles Oup and OSS', we have, by Euclid vi. 4—

$$\frac{up}{SS'} = \frac{uO}{OS} = \frac{5}{10} = \frac{1}{2}.$$

But SS' = 2 inches.

$$\therefore \frac{up}{2} = \frac{1}{2}, \text{ or } up = 1 \text{ inch.}$$

$$\therefore pp = uu + 2up = 2 + 2 = 4 \text{ inches.}$$

3. The intensity of illumination of a screen placed 6 feet from a given source of light is denoted by I . Find the intensity when the distance of the screen is increased to 9 feet.

Let I' denote the required intensity. Then, by Art. 9—

$$\frac{I'}{I} = \left(\frac{6}{9}\right)^2 = \left(\frac{2}{3}\right)^2 = \frac{4}{9}.$$

That is, $I' = \frac{4}{9} I$.

7. A circular uniform source of light, 10 cm. in diameter, is placed 1 metre in front of a spherical opaque body 5 cm. in diameter. Find the shortest distance from the latter at which a screen may be placed so as to have no umbra in the shadow cast upon it; also find the diameter of the penumbra in this position [Fig. 8 (c)].

8. A luminous sphere, 5 cm. in diameter, is placed 150 cm. from a disc of wood of 25 sq. cm. area. Find the dimensions of the umbra and penumbra cast on a screen 50 cm. behind the disc of wood. The line passing through the centre of the luminous sphere and the disc is perpendicular to the latter and to the screen.

9. In Fig. 6, $CO = 3$ metres, $OC' = 20$ cm., and the diameter of the aperture at O is 1 mm. Find the area of the circular spot of light at C' due to the pencil of light coming from C . If $AB = 2$ metres, find also the length of $A'B'$.

10. The intensities of two sources of light are in the ratio 9 : 16. Find the ratio of the distances at which they must be placed from a screen, in order to produce on it the same intensity of illumination.

11. The lines joining the points A , B , and C form an equilateral triangle. D is the middle point of BC . A screen is placed at A with its surface parallel to BC . Lights placed at B , C , and D are found to equally illuminate the screen at A ; compare their illuminating powers.

12. In Foucault's photometer (Fig. 11) $EL_1 : EL_2 :: a : b$. Find the relative intensities of L_1 and L_2 .

13. In Rumford's photometer (Fig. 12) $L_1 s$ is found to be 115 cm., and $L_2 s$ to be 201 cm. Compare the illuminating powers of L_1 and L_2 .

14. The intensities of two sources of light are in the ratio 4 : 9. If these sources are 200 cm. apart, where would a Bunsen's photometer be in accurate adjustment between them?

15. The distance between two incandescent lamps of 16 and 25 candle-power respectively is 6 feet. Show that there are two positions, on the line joining the lamps, at which a screen may be placed so as to receive equal illumination from each lamp; and determine these positions.

CHAPTER IV.

REFLEXION AT PLANE SURFACES.

14. WHEN a ray of light travelling in one medium, A, is incident on the surface of another medium, B, it is, in general, broken up into three parts.

1. A portion which is reflected from the surface of B, back into A, according to a certain law. This portion is said to suffer reflexion at the surface of B in accordance with the law of reflexion.

2. A second portion passes into B, and travels through that medium in a direction determined by another law. This portion is said to be refracted into the medium B in accordance with the law of refraction.

3. A third portion is scattered or diffused by the surface of B according to no definite law. The light thus scattered renders the surface luminous, and it is because of this scattering of light by the surfaces of non-luminous bodies that they become luminous in the presence of a self-luminous body (Art. 2).

When light is incident upon an opaque body no portion of the light is refracted, and the ratio of the quantities reflected and diffused depends on the nature of the surface of the body and on the angle at which the light falls on the surface. A rough, uneven surface scatters the greater portion of the light falling on it; but a smooth, highly polished surface reflects nearly all the incident light; also the more obliquely light falls upon any reflecting surface the greater is the proportion of reflected light. Since a surface is rendered visible by scattering the light incident upon it, it follows that a perfectly reflecting surface would be invisible.

15. Mirrors. Any good reflecting surface is a mirror. The term is, however, usually confined to polished surfaces of a definite geometrical form—*e.g.*, plane, spherical, cylindrical, etc.—but in optics only the first two are of importance. The oldest mirrors were of polished metal, and this form of reflector is now much used for optical purposes. The ordinary plane mirror consists of a sheet of plate glass backed by a thin layer of an amalgam of mercury and tin, which forms the reflecting surface. More recently, for scientific purposes, silvered *specula* have been employed as mirrors. These are formed of glass surfaces, of the required geometrical form, coated in front with a thin layer of silver which is very highly polished.

16. Definitions. The *normal* to a reflecting surface at any point is a line drawn at right angles to the tangent plane to the surface at that point. If the surface is plane, then the normal at any point is at right angles to the surface; and if spherical it is coincident in direction with the radius

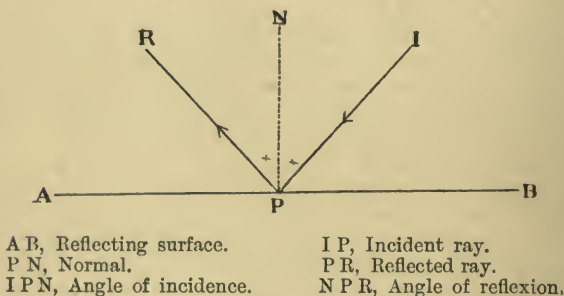


Fig. 14.

drawn to that point. The *angle of incidence* of a ray falling on the surface of a medium is the angle between the direction of the ray and the normal to the surface at the point of incidence. The *plane of incidence* is the plane containing the normal and the incident ray. The *angle of reflexion* is

the angle made by the reflected ray with the normal at the point of incidence. The *plane of reflexion* is the plane containing the normal and the reflected ray.

17. Laws of reflexion. When a ray of light is incident on a reflecting surface, it is reflected in accordance with two laws which may be thus formulated :—

1. The angle of reflexion is equal to the angle of incidence. ($\angle P N = \angle P R$, Fig. 14.)

2. The planes of incidence and reflexion are coincident.

These may be expressed as one law thus :—The angles of incidence and reflexion are in the same plane, and are equal to one another.

This law is established by experiment, and may be directly verified by means of the apparatus shown in the figure (Fig. 15). A graduated circle, fixed in a vertical plane, has a small mirror, *m*, attached horizontally at its centre, and carries two tubes, *T* and *T'*, having their axes directed towards the centre of the circle. These tubes travel round the circumference of the circle, and the position of their axes relative to the graduations is shown by a mark on the slide to which they are attached. The zero of the graduations is placed at the point where the normal to the mirror cuts the divided circle. A source of light is placed so as to send a beam of light down one of the tubes on to the mirror;

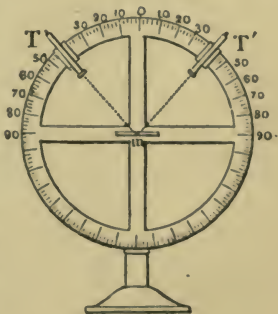


Fig. 15.

the other tube is then moved round until, on looking through it, the source of light can be seen reflected in the mirror. It will then be found that each of the tubes is at the same angular distance from the zero of the scale—that is, the angle of reflexion is equal to the angle of incidence, and the planes of incidence and reflexion are coincident with that of the divided circle.

18. Images. When a luminous body is viewed directly, pencils of light from every point on the body enter the eye, and thus the body is seen and its form defined. If, however, from any cause these pencils suffer change of direction, such that they actually come from, or appear to come from, an assemblage of luminous points other than the luminous surface of the body, this assemblage of luminous points is called the *image* of the luminous body. An image may be either *real* or *virtual*; in a real image the rays actually do come from the points of the image, but in a virtual image they only appear to do so. A real image differs from a luminous body in the fact that the latter emits light in all directions, whereas the former emits light only in the direction taken by the rays involved in its formation.

19. Reflexion of light from a luminous point at the surface of a plane mirror. Let L (Fig. 16) denote the position

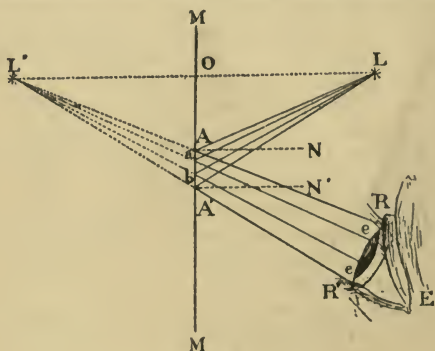


Fig. 16.

of the luminous point and MM that of the mirror. Consider the reflexion of any ray LA . Draw the normal AN at A ; then, according to the law of reflexion, the reflected ray will lie in the plane LAN , and its direction, AR , will be such that the angle RAN equals the angle LAN . Similarly, for the ray LA' , the reflected ray takes the direction

$A'R'$, such that $R'A'N'$ is equal to $LA'N'$. Now, to an eye placed at RR' , the pencil reflected from the portion AA' of the mirror will appear to come from a point L' at the intersection of RA and $R'A'$ (Art. 3). It can be shown that this point L' lies on the normal to the mirror, passing through L and at the same distance behind the mirror as L is in front of it. For, through L draw the normal LOL' , and let RA produced cut it at L' . Then, by Euclid i. 29, $LAN = OLA$,

$$\text{and } RAN = OL'A.$$

But, in accordance with the law of reflexion—

$$LAN = RAN;$$

$$\therefore OLA = OL'A.$$

\therefore in the triangles AOL and AOL' we have the angle $AOL =$ the angle AOL' and $OLA = OL'A$, and the side OA common. $\therefore OL = OL'$ (i. 26).

Similarly, it can be shown that any other reflected ray, if produced backwards, passes through L' ; and therefore, to an eye in front of the mirror, a virtual image of L is seen at L' . The image is virtual, because the rays by which it is seen do not actually come from L' , but, owing to the change of direction resulting from the reflexion at the surface of the mirror, they appear to do so.

It has here been proved, by assuming the truth of the law of reflexion, that the image of a luminous point is at the same distance behind the mirror as the point itself is in front of it. Hence, if this can be shown to be true experimentally, we get an indirect experimental proof of the law of reflexion (cp. Art. 17). This can be done in the following way:—Take a clean, polished plate of thin glass, and hold a pin, or other bright object, about a foot in front of it; an image of the pin, formed by the nearest polished surface of the glass, will, if the light is not too strong, be clearly seen on the opposite side of the plate, and another pin can easily be placed so as to coincide in position with this image. It will then be found, by direct measurement with a pair of compasses and a scale, that the distances of the image and the object from the reflecting surface are equal.

20. Reflexion of a convergent pencil incident on a plane mirror. The preceding article deals with the reflexion of a divergent pencil ($L A A'$), and shows that, after reflexion, it appears to diverge from a point at the same distance behind the mirror as that from which it originally diverged was in front of it. Similarly, if a convergent pencil $P L P'$ (Fig. 17), converging to a point L behind the mirror, be incident at $A A'$, it is reflected so as to converge to a point L' , such that $L O L'$ is normal to the mirror, and $O L = O L'$.

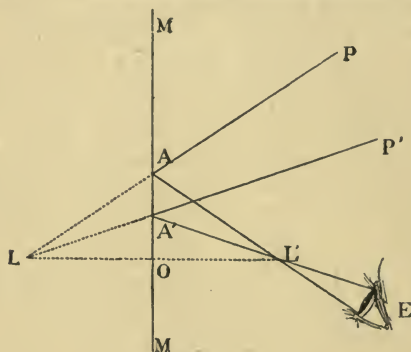


Fig. 17.

This can be proved in exactly the same way as the last case. An eye placed at E sees a *real* image at L' .

21. Image of a luminous object formed by a plane mirror. Let $A B$ (Fig. 18) represent a luminous object placed in front of the mirror $M M$. As in Art. 19 the image of A is formed at A' , such that $A O A'$ is normal to $M M$ and $A' O$ equal to $A O$. Similarly, the image of B is formed at B' , such that $B O B'$ is normal to $M M$ and $B' O$ equal to $B O$. For all points of the object intermediate between A and B images are formed at corresponding points between A' and B' , and thus a complete image of the object is formed at $A' B'$.

A more elaborate construction is sometimes given for

determining the position of an image formed by a plane mirror, and, as the method is general and applicable to spherical mirrors, we shall briefly notice it.

It is based on the fact that, owing to symmetry, the focus of any two reflected rays, coming from any point of the image, is also the focus of all the reflected rays coming from the same point; hence, the intersection of any two reflected rays determines the point on the image from which they diverge, or appear to diverge. For plane mirrors, the two rays chosen are AO (Fig. 18),

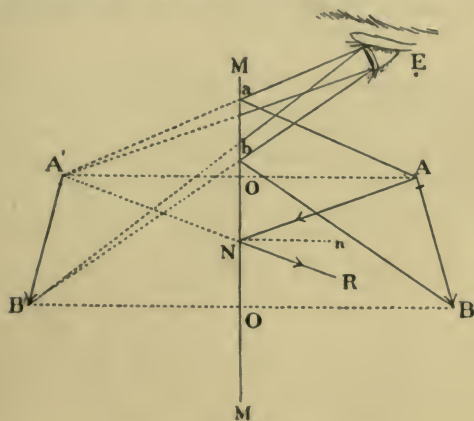


Fig. 18.

incident along a normal to the mirror, and any other ray incident in any other direction, such as AN . The ray AO is reflected back along its original path, and AN is reflected along NR , making the angle of reflexion RNn equal to the angle of incidence ANn , and the image of A is formed at A' , the virtual focus of the reflected rays OA and NR . Similarly, the image of any other point B is obtained, and the images of intermediate points assumed to lie on the line $A'B'$, and hence $A'B'$ is said to be the image of AB . When the form of the image is more complex than that considered here, the images of

a number of points, sufficient to determine the complete image, must be obtained.

An eye placed at any point E (Fig. 18), in front of the mirror, sees the image $A'B'$ by light reflected from the portion ab of the surface of the mirror, and the actual path of the *extreme* rays is shown by the lines AaE , BbE . It is evident from this that, in order that any point of an image may be seen, the line joining this point to the eye must cut the surface of the mirror, and the portion of the surface at which, by reflexion, an image is seen, is that portion which is intercepted by the cone having the eye at its apex and the image as its base.

22. Path of rays by which an image is seen. Let L' (Fig. 16) represent the image of a luminous point L formed by the mirror MM , and imagine an eye placed at E . Draw lines joining L' to the extremities ee of the aperture of the eye, and cutting the mirror at a and b ; then join L to a and b , and the lines Lae and Lbe define the pencil of light by which L' is seen (cp. Art. 21). Each point of the

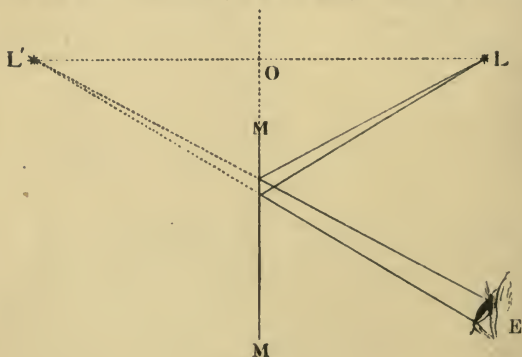


Fig. 19.

image of a luminous object is seen in the way just described, and the extreme rays bounding the collection of pencils reaching the eye are determined in the way indicated at AaE , BbE in Fig. 18.

In connection with this question it is important to notice what must be the position of an object relative to a mirror in order that an image may be formed by that mirror. Let MM (Fig. 19) represent a mirror; then, if an object L be placed anywhere in front of the plane passing through MM , an image of that object will be formed behind this plane, at a point L' , such that LOL' is normal to the plane and $LO = L'O$. This can be proved in the same way as the proposition of Art. 19; the figure, which corresponds to Fig. 16, shows the necessary construction, and also the path of the rays by which an eye placed at E is able to see the image L' .

23. Lateral inversion. When the image of the face is seen in an ordinary looking-glass, we know that the image of the right eye forms the left eye of the reflected face, while the image of the left eye forms its right eye. This is a particular instance of a result of reflexion known as lateral inversion. It does not affect the appearance of objects which are bi-laterally symmetrical; but with non-symmetrical objects, such as printed or written characters, the effect is sufficiently evident and well known.

24. Deviation. When a ray of light is turned out of its original course it is said to suffer deviation, and the

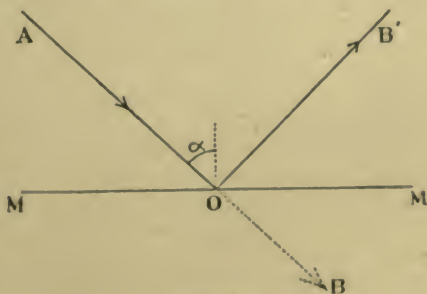


Fig. 20.

angle between its initial and final direction determines the amount of this deviation. The deviation due to a single reflexion at a plane surface is easily determined. Let AO (Fig. 20) be incident on the surface MM at an angle a to the normal. Then, since the initial direction of the ray is represented by AB , and its final direction by OB' , the deviation is evidently given by the angle BOB' . But $BOB' = 180 - AOB' = 180 - 2a$.

25. Reflexion from a rotating mirror. Let NA (Fig. 21) represent a ray incident normally on the mirror MM . If the position of the mirror remain unchanged, then NA will be reflected back along AN ; but if MM be rotated, in the direction shown by the arrows, round an axis at A ,

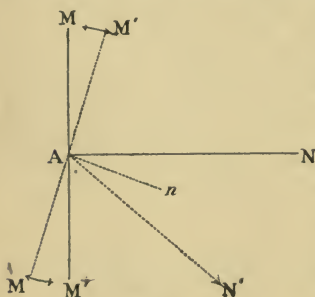


Fig. 21.

into the position $M'M'$, then NA will be reflected along AN' according to the law of reflexion. Now the angle $NA n = MAM'$. $\therefore NANN' = 2NA n = 2MAM'$. But $NANN'$ is the angle through which the reflected ray has been rotated by the rotation of the mirror through MAM' . Hence, if a mirror be turned through an angle a , the reflected ray is rotated through

an angle $2a$; that is, the reflected ray rotates twice as rapidly as the mirror from which it is reflected.

26. Reflexion at plane surfaces inclined to one another. Before considering particular cases of special interest, it will greatly simplify matters to notice the general principles applicable to all cases. Imagine an object A , placed between two mirrors, M_1 and M_2 , inclined to each other at any angle. An image of A will be formed by each mirror; and, if the image formed by M_1 lie in front of M_2 —that is, if it is anywhere in front of the plane in which this mirror lies (Art. 22)—then an image of this image will be formed

by M_2 . Similarly, if the image formed by M_2 lie in front of M_1 , then an image of this image is formed by M_1 . These are said to be images of the second order. In precisely the same way, if this second pair of images are suitably placed, a third pair (of the third order) may be formed, and so on. This multiplication of images stops when a pair is formed in the space behind both mirrors—that is, within the angle vertically opposite to that in which the object is placed. We shall now consider a few special cases.

(1) **Parallel mirrors.** Let M_1 and M_2 (Fig. 22) represent two parallel mirrors, and A an object placed between them. It is evident that since the mirrors are parallel no image can be formed behind both, and hence every pair of images gives rise to another pair, and thus an infinite series of images may, theoretically, be formed. Through A draw $N_1 N_2$ normal to both the mirrors, and produce it indefinitely on both sides. In obedience to the law of reflexion all the

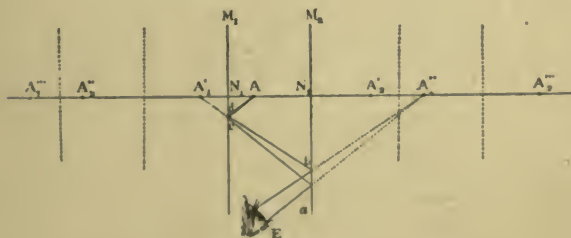


Fig. 22.

images must lie on this line, and their positions on it will depend on the position of A between M_1 and M_2 , and on the distance between these mirrors. Consider first the reflexion from M_1 ; an image of A is formed at A_1' on the normal through A , and so placed that $A_1' N_1$ equals $A N_1$. Similarly an image of A_1' is formed by M_2 at A_1'' in such a position that $A_1'' N_2 = A_1' N_2$; A_1'' in turn gives rise to

A_1''' by reflexion at M_1 , and so on. In the same way, beginning with the first reflexion at M_2 , the images A_2' , A_2'' , etc., are formed by successive reflexion at M_2 and M_1 . In Fig. 22 the position of the images up to the third order are shown, and, to distinguish them, the suffix attached to A denotes the mirror at which the *first* reflexion took place, and the dashes indicate the order of the image. Thus the series A_1' , A_1'' , A_1''' . . . is formed by successive reflexions from M_1 and M_2 , beginning with reflexion at M_1 ; similarly the series A_2' , A_2'' , A_2''' . . . is formed by successive reflexions from M_2 and M_1 , beginning with M_2 . The members of each series are so related that any one may be considered as the image of the one immediately preceding it: for example, A_1''' may be considered as the image of A_1'' formed by the mirror M_1 , and consequently $A_1'''N_1$ equals $A_1''N_1$.

To determine the path of rays by which any image is seen, the following construction should be employed. Let it be required to find the path of the rays by which an eye at E sees the image A_1'' . First trace this image back to A ; A_1'' is an image of A_1' , which is itself an image of A . Now join the extremities of the aperture of the eye to A_1'' by lines cutting M_2 at a and b , and mark the real parts of this path, which, since the rays cannot penetrate the mirror, must lie between the eye and ab . Next join a and b to A_1' by lines cutting M_1 in c and d , and mark ac , bd as the real portions of this path. Then finally join c and d to A , and the twice reflected pencil passing from A to E indicates the required path. From this it is evident that an image of the *second* order, A_1'' , is seen by *two* reflexions, and therefore an image of the n^{th} order would be seen by n reflexions. The mirror from which the last reflexion takes place—that is, the mirror in which the image is seen—depends upon whether n is odd or even. In either series of images the *odd** members are seen by reflexion from the mirror at which the *first* reflexion takes place, while the even numbers are seen in the other mirror. At each

* That is, the 1st, 3rd, 5th, etc.

reflexion there is some loss of light, depending in amount on the polish of the reflecting surface; and, as a consequence, the higher the order of any image the fainter it appears, until finally it becomes too faint to be visible.

(2) **Mirrors inclined at right angles.** Let OM_1 and OM_2 (Fig. 23) represent two mirrors at right angles to each other, and A an object placed between them. Then an image A_1' is formed by OM_1 and A_2' by OM_2 . But A_1' lies in front

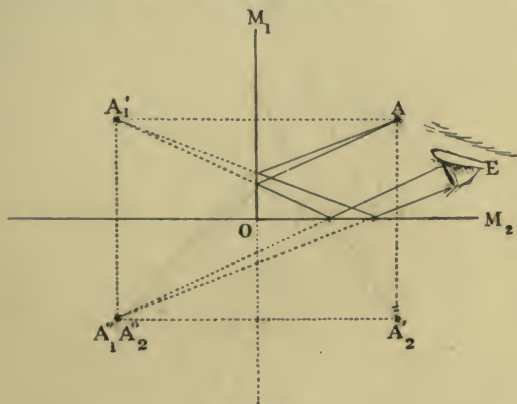


Fig. 23.

of OM_2 , and therefore an image A_1'' is formed by that mirror. Also, A_2' is in front of OM_1 , and therefore gives rise to the image A_2'' , which from the geometry of the figure is evidently coincident with A_1'' . An eye placed anywhere within the proper limits (Art. 21) sees three images, A_1' , A_1'' , or A_2'' , and A_2' , at the three corners of the right-angle $A_1'A_2'$. Both the images A_1'' and A_2'' cannot be seen at the same time; an eye placed within the angle M_1OA sees the image A_2'' ; while one placed in M_2OA sees A_1'' . The figure shows the path of the rays by which the image A_1'' may be seen by an eye placed at E . The method of determining this path is indicated by the associated dotted lines, and is exactly similar to that explained above for parallel

mirrors. The actual path of the rays necessarily lies within the angle $M_1 O M_2$.

(3) **Mirrors inclined at any angle.** Let $O M_1$ and $O M_2$ (Fig. 24) represent two mirrors inclined at the angle $M_1 O M_2$ and A an object placed between them. With O as centre, describe a circle passing through A and cutting $O M_1$ and $O M_2$ in N_1 and N_2 respectively. Then all the images of A must lie in the circumference of this circle. For consider the image A_1' ; according to the law of reflexion it is so placed that $A n_1$ equals $A_1' n_1$, and AA_1' is at right angles to

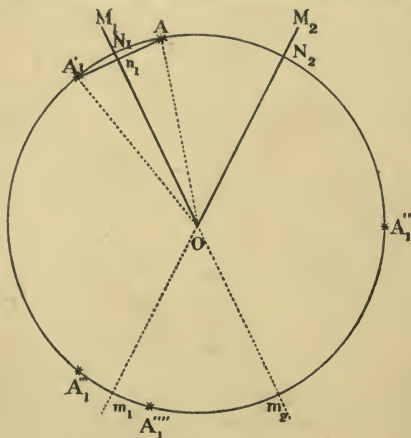


Fig. 24.

$O M_1$. Hence, in the triangles $O A n_1$ and $O A_1' n_1$ we have $A n_1$ equal to $A_1' n_1$, $n_1 O$ common, and the angle $A_1' n_1 O$ equal to the angle $A n_1 O$; therefore $O A_1'$ equals $O A$ (Euc. i. 4), and A_1' lies on the circumference of the circle passing through A . Similarly for any other image.

The successive formation of images is exactly similar to that described for parallel mirrors; each mirror gives rise to a separate series of images, but in this case the number of images that can be formed is limited, the last member of each series being that formed within the angle $m_1 O m_2$.

Fig. 24 shows the positions of the members of the series formed by first reflexion from M_1 . In general, the last members of the series, formed on the arc $m_1 m_2$, have different positions; but when the angle $M_1 O M_2$ is an aliquot part of 360° , then these images are coincident. Figs. 23 and 25 illustrate this; in Fig. 23, $M_1 O M_2$ (90°) is one-fourth of 360° , and the images A''_1 and A''_2 are coincident; similarly, in Fig. 25, $M_1 O M_2$ (60°) is one-sixth of 360° , and the images A'''_1 and A'''_2 are coincident. From Fig. 25 it will be seen that the number of images formed is 5 when $M_1 O M_2$ is *one-sixth* of 360° —that is, if the angle $M_1 O M_2$ be $1/n^{\text{th}}$ of 360° , then the number of images formed is $(n-1)$. Or, if $M_1 O M_2$ be denoted by θ , then, when θ is an aliquot part of 2π , the number of images formed is given by $\left(\frac{2\pi}{\theta} - 1\right)$.

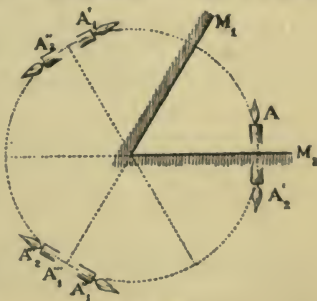


Fig. 25.

This is the principle of the **kaleidoscope**. Two long narrow mirrors, inclined to each other at an angle of 60° , are placed in a slightly longer tube. One end of the tube is closed by a metal disc, pierced at the centre, with a hole through which the observer looks; at the other end a plate of clear glass fits into the tube close up to the mirrors; and a short distance beyond it, at the end of the tube, is a similar plate of ground glass. Between these two glass plates little pieces of coloured glass, etc., are loosely placed, and, with their images, form beautiful and symmetrical patterns visible to an eye placed at the other end of the tube. On rotating the tube the pieces of glass change position, and thus the pattern seen is continually changing. Sometimes three mirrors are employed, the arrangement being such that the cross section of the three is an equilateral triangle. Each pair of plates acts in the way described above, so that the arrangement gives rise to intricate but symmetrical patterns, which are capable of giving material aid to designers.

CALCULATIONS.

28. ALL problems on reflexion at plane surfaces are, more or less, geometrical deductions, involving a knowledge of the laws of reflexion in addition to the usual geometrical propositions.

The results of Art. 26 are not of very great importance, but the simple case where θ is an aliquot part of 360° should be remembered. In this case the number of images formed is $\left(\frac{2\pi}{\theta} - 1\right)$.

Note.—In preparation for the work of the next chapter the reader should notice the following points :—

1. The results of Euclid vi. 3, A, and 4.
2. The meaning of the terms *infinite* and *infinity*. A quantity becomes *infinite* when its value becomes greater than any value we can assign to it. If the value of any quantity q is infinite, this is expressed by writing $q = \infty$.

The term *infinity* will be best understood from its use in the statement that parallel straight lines meet at infinity. If any straight line OA be produced to A' , in the direction $O A$, until it is of infinite length, the point A' will be at infinity.

3. Consider the ratio $\frac{a}{x}$. If x becomes infinite, the ratio becomes $\frac{a}{\infty}$, and the value of this expression is zero. That is—

$$\frac{a}{\infty} = 0$$

where a is *any* finite quantity.

4. The sine of any angle is equal to the sine of its supplement. That is—

$$\sin \alpha = \sin (180 - \alpha).$$

This is readily seen from a figure.

EXAMPLES II.

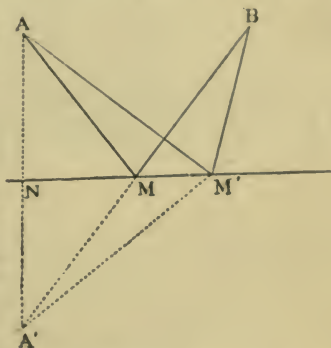
1. A ray of light starts from A, meets a plane reflecting surface at M, and is reflected to B. Prove that A M B is the shortest possible path from A to B by way of the mirror.

If A M B be not the shortest path, let *any* other path A M' B be shorter. Draw A N A' normal to the mirror, and produce B M to meet A A' in A'.

Then, since $AN = A'N$ we have, by Euclid i. 4, $AM = A'M$ and $AM' = A'M'$.

But $A'M' + M'B > A'B > A'M + MB$ (Euc. i. 20).

$\therefore AM + MB > AM' + M'B$.
Q.E.D.



4. A small object is placed between two parallel mirrors as in Fig. 22. The distance between the mirrors is 6 inches, and the object is placed 2 inches from one of them. Find the distances between the corresponding members of the two series of images formed; also the distances between the odd members of each series, and between the even members of each series.

5. The sun is 30° above the horizon, and you see his image in a tranquil pool. What, in this case, is the angle of incidence and reflexion?

6. A man, 6 feet high, sees his image in a plane mirror hung vertically. The top of the mirror being 6 feet from the ground, determine its smallest length in order that the man may see his full-length image in it.

7. Find the deviation produced by reflexion at a plane mirror, when the angle between the incident and reflected rays is 80° .

9. Show, that if a ray of light be incident at any angle, on one of two mirrors inclined at right angles to each other, then the ray is reflected from the second mirror in a direction parallel to its original direction.

10. A mirror revolves about a horizontal axis parallel to its surface. Show how to find if the reflecting surface is accurately parallel to the axis of revolution.

CHAPTER V.

REFLEXION AT SPHERICAL SURFACES.

29. Preliminary definitions. A spherical mirror, AA' (Fig. 27), is usually a very small segment of a spherical surface, and may be either convex or concave, according

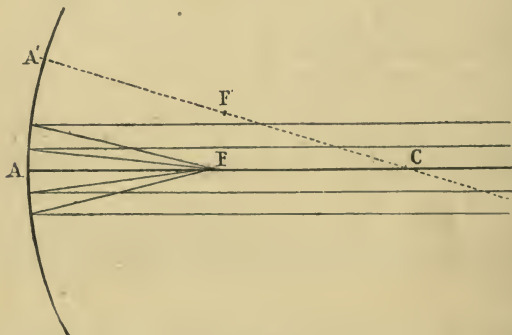


Fig. 27.

to the curvature of the reflecting surface. The centre, C , of the spherical surface of which the mirror is a part is called the *centre of curvature* of the mirror. The line CA joining the centre of curvature and the central point, A , of the mirror is the *principal axis* of the mirror, the point A being sometimes called the *pole* or *centre of the face*. Any other line CA' drawn through C and cutting the mirror is called a *secondary axis*, and is, like the principal axis, a normal to the mirror.

When a parallel pencil of light is incident on a spherical mirror, in a direction parallel to the principal axis, the reflected pencil converges to a point F on the principal axis. This point is called the *principal focus* of the mirror, and the distance, AF , between the principal focus and the pole of the mirror is termed the *focal length* of the mirror. If the pencil is incident parallel to a secondary axis, the reflected rays are, in a similar way, brought to a focus at a point F' on that axis. In the case of spherical mirrors, it is not strictly true to say that the reflected pencils meet accurately at a point; if the pencil is small, this is approximately the case, but with large pencils the outer rays are reflected to points nearer the mirror than the others. This irregularity of reflexion from a spherical surface is called *spherical aberration*.

30. Construction for reflected ray. Let PQ be any ray incident at Q on a spherical mirror [concave Fig. 28 (A), or convex Fig. 28 (B)]. At Q draw the normal, QN , to the reflecting surface, by joining CQ and producing it if

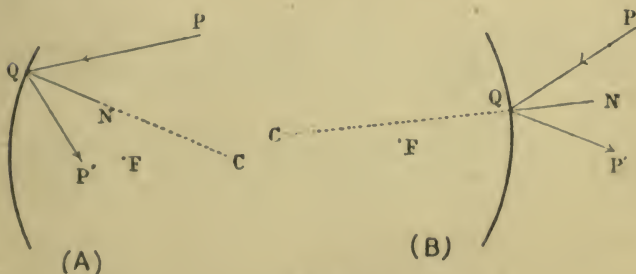


Fig. 28.

necessary. Then, in accordance with the law of reflexion, the reflected ray QP' is obtained by drawing QP' in such a direction that the angle of reflexion $P'QN$ is equal to the angle of incidence PQN .

From this it is evident that a ray incident along a normal is reflected back along the path by which it came. Also, from Art. 29, a ray incident parallel to the principal axis is reflected through the principal focus. These two particular cases of reflexion should be carefully remembered.

31. Position of principal focus. Let PQ (Fig. 29) be a ray incident on the concave mirror AQ , in a direction parallel to the principal axis CA . Then PQ is reflected through the principal focus F , and the angle PQC is equal

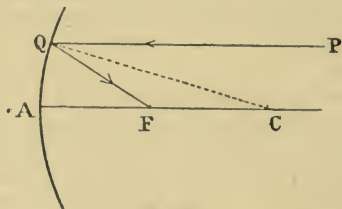


Fig. 29.

to the angle FQC . But the angle PQC is equal to the angle FCQ (Euc. i. 29); therefore $FQC = FCQ$, and therefore $FQ = FC$.

Now, if AQ is small, FQ is approximately equal to FA , and therefore FC equals FA (approximately). That is, the principal focus F is midway between the pole A and the centre of curvature C ; and, if AF be denoted by f and AC by r , we have $f = \frac{r}{2}$; or the focal length of a spherical mirror, for rays incident on a small portion of its surface near the pole, is equal to half the radius of curvature of that mirror.

The proof here given applies directly to a concave mirror, but an exactly similar proof is applicable in the case of a convex mirror. The student should draw the figure and follow out the steps of the proof for himself.

32. Conjugate foci.* Let P (Fig. 30) represent the position of a luminous point on the principal axis of the

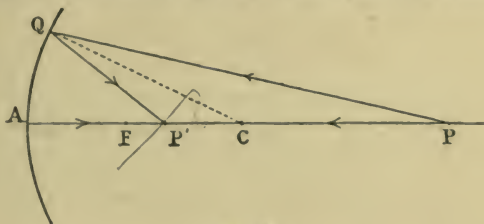


Fig. 30.

concave mirror A Q. Then, Art. 21, the image of P will be formed at the intersection, after reflexion, of any two rays coming from A. Consider the rays P A incident along the normal to the mirror, and P Q incident at Q; P A is reflected back along A P and P Q is reflected along Q P', making the angle of reflexion P' Q C equal to the angle of incidence P Q C. Let the reflected rays A P and Q P' intersect at P'; then P' is the image of P, and lies on the principal axis of the mirror. Also, since P Q C = P' Q C, then—

$$\frac{Q P'}{Q P} = \frac{P' C}{C P}. \quad (\text{Euclid vi. 3.})$$

But, if A Q is small, then P Q = P A, and P' Q = P' A, and—

$$\frac{A P'}{A P} = \frac{P' C}{C P}.$$

If now A C be denoted by r , A P by u , and A P' by v , then P' C = A C - A P' = $r - v$, and C P = A P - A C = $u - r$. And the above proportion becomes—

$$\frac{v}{u} = \frac{r - v}{u - r}.$$

$$\therefore u(r - v) = v(u - r).$$

$$\therefore u r + v r = 2 u v.$$

Dividing by $u r v$ then—

$$\frac{1}{v} + \frac{1}{u} = \frac{2}{r}.$$

* The reader should refer to Euclid vi. 3 and A before reading this article.

But, by Art. 31—

$$f = \frac{r}{2} \quad \therefore \frac{2}{r} = \frac{1}{f}, \text{ and}$$

$$\therefore \frac{1}{v} + \frac{1}{u} = \frac{1}{f}, \quad (1).$$

where u denotes the distance of the luminous point P from the pole of the mirror, v denotes the distance of the image of P from the pole of the mirror, r denotes the radius of curvature of the mirror, and $f(=\frac{r}{2})$ denotes the focal length of the mirror.

The relation thus obtained is of great importance. It will be noticed that, in the case here considered, all the distances involved are measured in the same direction from A . When this is not the case, it is necessary to adopt some convention as to sign. The most general convention, and the one adopted throughout this book, is *to consider all distances measured in a direction opposed to the incident light as positive, and distances measured in the same direction as the incident light as negative*. With this convention the formula $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$ is applicable to all cases of reflexion at spherical mirrors.

The points P and P' connected by this relation are said to be *conjugate foci*, because of the fact that either point may be considered as the image of the other. From the construction it is evident that the image of a luminous point at P' would be formed at P , just in the same way as the image of P is formed at P' . This may be illustrated experimentally by means of a candle and a concave mirror. If the flame of the candle be placed, at any point beyond C , on the principal axis of the mirror, an image of the flame will be seen between C and the mirror. The position of this image can be marked by adjusting the position of a needle until it appears to coincide with the image. It will then be found that if the candle flame be placed at the point marked by the needle, the image will be seen at the point originally occupied by the flame. If the luminous point P is not on the principal axis, then its conjugate focus, P' ,

will be on the secondary axis passing through P, and, distances being measured along this axis, the relation, $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$ can be established in the way explained above. In fact, the two cases are identical, for the geometrical relations of a secondary axis to a spherical mirror are exactly the same as those of the principal axis.

33. Relative position of conjugate foci. In the preceding article it has been shown that the formula $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$ holds good for all cases of reflexion at a spherical surface. By a general discussion of this formula it is possible to determine the position of P' for any given position of P. For example, if u be infinite,—that is, if the incident light be parallel,—then we have—

$$\frac{1}{v} + \frac{1}{\infty} = \frac{2}{r}.$$

Therefore, since $\frac{1}{\infty} = 0$

$$\frac{1}{v} = \frac{2}{r} \text{ or } v = \frac{r}{2}.$$

This means, that if a pencil of parallel light be reflected at the spherical surface, its focus, after reflexion, is on the axis parallel to the incident light at a point whose distance from the mirror is equal to half the radius of curvature of the mirror (Art. 31).

In addition to the above, the following general rule will be found of great use in determining the motion of the image corresponding to any given motion of the object along an axis of the mirror:—*When an image is formed by reflexion, any motion of the object, in a given direction along an axis of the mirror, causes motion of the image in an opposite direction along the same axis.**

By the application of this rule we may trace the motion of P' as P travels from infinity up to a spherical mirror.

* In the case of a plane mirror, any normal to its surface may be considered as an axis.

Thus, considering first a concave mirror, when the incident light is parallel—that is, when P is at infinity in front of the mirror—P' is at F (Fig. 32). As P travels from infinity up towards C, P' travels, *in the opposite direction*, from F towards C. At C, P and P' meet, and image and object coincide. As P travels from C to F, P' travels, in the opposite direction, from C to infinity, and the initial relations of P and P' are now reversed. As P further travels from F to A, P' travels, *in the opposite direction*, from infinity to A—that is, P', after disappearing at infinity *in front* of the mirror, reappears from infinity *behind* the mirror and travels up to A.

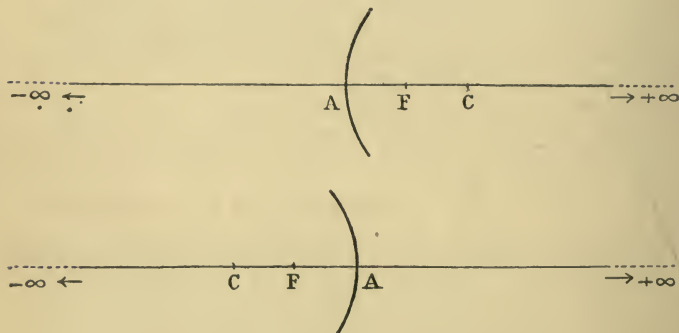


Fig. 32.

The case of a convex mirror is simpler. The principal focus, F (Fig. 32), is *behind* the mirror; hence, as P travels from infinity up to the mirror, P' travels, in the opposite direction, from F up to the mirror.

Now, if P be a real luminous point, or small object, such as a candle flame, it can evidently travel no further than A, and thus we have traced all possible positions of the image of a real luminous point placed anywhere in front of a spherical mirror. The following positions should be noted :—

I. Concave mirror.

1. Luminous point between $+\infty$ and C. Real image between F and C.
2. Luminous point at C. Real image at C.
3. Luminous point between C and F. Real image between C and $+\infty$.
4. Luminous point between F and A. Virtual image between A and $-\infty$.
5. Luminous point at A. Image at A.

II. Convex mirror.

1. Luminous point between A and $+\infty$. Virtual image between A and F.

34. Formation of images by spherical mirrors. When a luminous object is placed in front of a spherical mirror an image is formed, which may be *real* or *virtual* according to the circumstances of the case. If real, the image is formed in front of the mirror, and can be received on a screen; but if virtual, it appears to be behind the mirror, and cannot be received upon a screen.

The following is a general construction for determining the image of an object formed by a spherical mirror. Let A B (Figs. 33, 35, 36) represent an object placed in front of

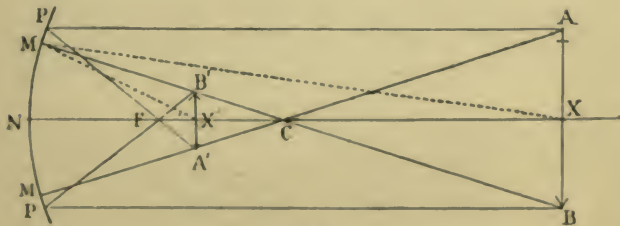


Fig. 33.

the mirror M M. Consider the ray A M coming from A and incident on the mirror normally at M. Its direction is obtained by joining A C, and, if necessary, producing the

line to cut the mirror in M. The reflected ray M A travels back along the path of the incident ray (Art. 30), and the image of A lies somewhere on this path. Again the ray A P, drawn parallel to the principal axis, is reflected along P F (Art. 30), and the image of A lies on this line also. Hence the image of A is found at A', the intersection of the lines M A and P F. Similarly, an image of B is formed at B', and images of points lying between A and B are formed at corresponding points between A' and B', and therefore A' B' is the complete image of A B.

In connection with the formation of images the following four points have to be considered :—

1. *Relative position of image and object.* This has been fully considered in preceding article; the reasoning there employed is applicable whether the luminous point P be an isolated point, or a point on an object of finite size.

2. *Whether the image is real or virtual.* Whenever the image appears behind the mirror, it must necessarily be virtual; hence, it is only necessary to know the position of an image to decide whether it is real or virtual.

3. *Whether the image is inverted or erect.* It is evident, from Fig. 33, that when object and image are on opposite sides of C, the latter is inverted because of the crossing of the rays passing through C. Hence, if the relative positions of object and image are known, this point is easily decided.

4. *Relative size of image and object.* Let A B (Fig. 33) represent an object, and A' B' its image formed by the mirror M M. Now in the triangles A' B' C and A B C we have—

$$\frac{A' B'}{A B} = \frac{C X'}{C X}.$$

That is—

$$\frac{\text{Image}^*}{\text{Object}} = \frac{\text{Distance of image from C}}{\text{Distance of object from C}} = \frac{c'}{c}.$$

* This ratio, $\frac{\text{image}}{\text{object}}$, is sometimes called the *magnification*, and must be understood to mean—

$$\frac{\text{Linear dimension of image.}}{\text{Corresponding linear dimension of object.}}$$

This proportion applies to linear dimensions only; for relative area we have—

$$\frac{\text{Area of image}}{\text{Area of object}} = \left(\frac{c'}{c}\right)^2.$$

Also, from the triangle $X' M X$ (Fig. 33), we have—

$$\frac{X' C}{C X} = \frac{M X'}{M X}.$$

But $\frac{X' C}{C X} = \frac{A' B'}{A B}$; and, if $N M$ be small, $\frac{M X'}{M X} = \frac{N X'}{N X}$ (approximately).

$$\therefore \frac{A' B'}{A B} = \frac{N X'}{N X} = \frac{v}{u}.$$

That is—

$$\frac{\text{Image}}{\text{Object}} = \frac{\text{Distance of image from mirror}}{\text{Distance of object from mirror}}.$$

Also—

$$\begin{aligned} \frac{1}{v} + \frac{1}{u} &= \frac{1}{f} \\ \text{or } \frac{1}{v} &= \frac{1}{f} - \frac{1}{u} = \frac{u-f}{uf} \\ \therefore v(u-f) &= uf. \\ \therefore \frac{v}{u} &= \frac{f}{u-f}. \end{aligned}$$

That is—

$$\frac{\text{Image}}{\text{Object}} = \frac{f}{u-f}.$$

Thus we have—

$$\frac{\text{Image}}{\text{Object}} = \frac{c'}{c} = \frac{v}{u} = \frac{f}{u-f}.$$

In this way we get three different but not independent relations between the linear dimensions of the image and object. These are—

- (1) $\frac{\text{Image}}{\text{Object}} = \frac{\text{Distance of image from mirror}}{\text{Distance of object from mirror}} = \frac{v}{u}$
- (2) $\frac{\text{Image}}{\text{Object}} = \frac{\text{Distance of image from } C}{\text{Distance of object from } C} = \frac{c'}{c}.$
- (3) $\frac{\text{Image}}{\text{Object}} = \frac{\text{Focal length of mirror}}{\text{Distance of object from focus}} = \frac{f}{u-f}.$

It thus appears that if we know the positions of the object and its image we can completely determine the nature of the image. The results of Art. 33, as there summarised, are therefore of great importance, and for this reason we give below, with figures, the cases for a luminous object of finite size, corresponding to I., 1, 2, 3, 4, and II. 1 of that article.

I. Concave mirror.

(1) Object between C and infinity in front of mirror. The image lies between C and F, and is *real, inverted, and diminished*. (Fig. 33.)

(2) Object C B at C.

The image, C B', at C, is *real, inverted, and of same size* as object. (Fig. 34.)

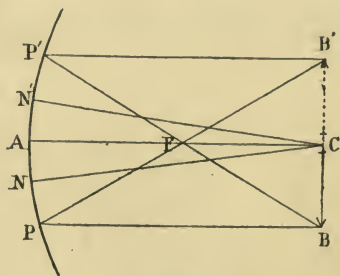


Fig. 34.

The construction for this image should be noticed. Any two rays C N, C N' coming from C are normal to the mirror, and therefore, on reflexion, again intersect at C. That is, the image of C is formed at C. The ray B P, parallel to principal axis, is reflected through F, along P F B'. Also B F P', passing through F, is reflected parallel to the principal axis along P' B'. Hence, an image of B is formed at B', the intersection of the reflected rays P F B' and P' B'. This construction may, if preferred, be applied to all cases; here it is necessary because the ray from B, passing through C, does not fall on the mirror.

(3) Object between C and F. Image between C and

infinity in front of the mirror. If $A'B'$ of Fig. 33 be supposed to represent the object, then AB represents its image, and the figure illustrates the case we are now considering. The image is *real*, *inverted*, and *magnified*.

(4) Object between F and A . The image is *behind* the mirror, between infinity and A , and is *virtual*, *erect*, and *magnified*. (Fig. 35.)

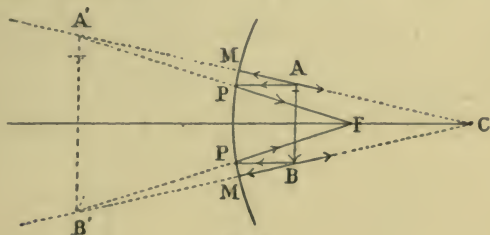


Fig. 35.

In the limit, when the object is at A , the image is also at A , and coincides with the object in position and size (Art. 33, I. 5).

II. Convex mirror.

(1) Object in front of mirror between infinity and A .

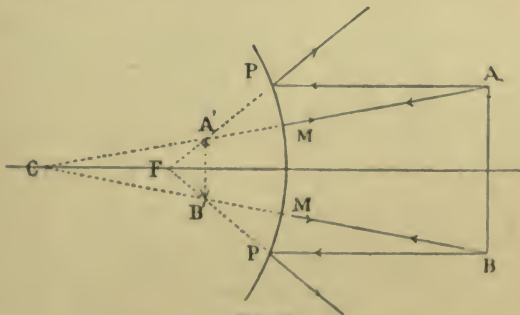


Fig. 36.

The image lies between F and A , and is *virtual*, *erect*, and *diminished*. (Fig. 36.)

35. The optical bench. This apparatus is of such frequent use in optical measurements that it is advisable, at this stage, to consider briefly its construction and method of use. In one of its simplest forms the optical bench consists of a thick base board (B B, Fig. 37) of well-seasoned wood, about three metres long, and having a deep wide groove running along the middle of its upper face. The edges of this groove are not vertical, but are obliquely cut in the way shown at *s*, Fig. 37. A scale, showing centimetres and millimetres, is cut parallel to the groove in such a way that the edge of the groove is also the measuring edge of the scale. A set of uprights, constructed to hold suitably mounted* lenses, mirrors, candles, screens, etc., are fitted into small

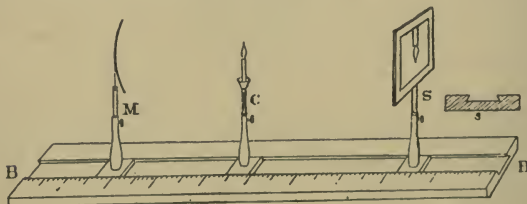


Fig. 37.

base boards, which are so made that they can be pushed into the groove in B B at one end, and moved along to any position on the bench. This position is indicated, with reference to the scale of the bench, by means of a fine index line cut on the base of the upright in the plane of its vertical axis.

This piece of apparatus may be used for the verification of the formula $\frac{1}{v} + \frac{1}{u} = \frac{1}{f} = \frac{2}{r}$ in the case I. 3, Art. 34.

For this purpose a concave mirror, the radius of curvature of which is known, is fixed in one of the uprights M at one end of the bench, and a candle, fixed in another upright, C,

* The mounting of any object should be effected in such a way that the point, to or from which measurements are to be made, is on the axis of the upright.

is placed in front of the mirror, between its principal focus and centre of curvature. A screen of white unglazed paper, mounted on a light frame, is fixed in a third upright, S, at the other end of the bench. The positions of S and C are now adjusted until a clearly defined inverted image of the candle flame is seen on the screen. The distances between M and S, and M and C, are then read off on the scale.

These distances give respectively v and u , and if $f (= \frac{r}{2})$ is known, they may be used in verification of $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$, or, in f be unknown, they can be used for its determination by calculation from the same formula.

The optical bench may also be conveniently used for photometric measurements with Bunsen's photometer (Art. 12), which, mounted on an upright placed between two other uprights carrying the lights to be compared, is readily adjusted in the right position, and the required distances are at once read off on the scale.

36. Experimental determination of the radius of curvature and the focal length of a concave mirror. The simplest method of making this determination is by application of the fact, referred to in Art. 34, that when an object is placed at the centre of curvature of a concave mirror, the image is also at the centre, but in an inverted position. Let a short needle be fixed, vertically and point upwards, in a clip, and placed in front of the mirror, at such a height that the point of the needle is on the principal axis of the mirror.

Unless the needle be placed too close to the mirror, an inverted image may be seen by an eye placed near the principal axis, at some distance from the mirror. By adjusting the position of the needle its point may be made to coincide with that of the inverted image. The point of the needle is now at the centre of the mirror, and hence the radius of curvature is obtained by measuring the distance between the pole of the mirror and the point of the needle. The focal

length is equal to half this distance. If the distance is small, it may be measured by means of a pair of compasses and a scale; but, if large, a wooden rod, pointed at both ends and of adjustable length, may be used, or the mirror and needle may be fixed in two of the uprights of the optical bench, and the measurements made as described above (Art. 35).

CALCULATIONS.

37. THE formulæ of importance in the preceding chapter are—

$$1. \quad \frac{1}{v} + \frac{1}{u} = \frac{1}{f} = \frac{2}{r}. \quad (\text{Art. 32.})$$

Distances measured from the pole of the mirror.

$$3 (a). \quad \frac{\text{Image}}{\text{Object}} = \frac{v}{u}.$$

Distances measured from the pole of the mirror.

$$3 (b). \quad \frac{\text{Image}}{\text{Object}} = \frac{c'}{c}.$$

Distances measured from the centre of curvature.

$$3 (c). \quad \frac{\text{Image}}{\text{Object}} = \frac{f}{u-f}.$$

Distances measured from the pole of the mirror.

Distances measured in a direction **opposed** to that of the incident light are considered **positive**, and those measured in the **same** direction as the incident light are considered **negative**.

This convention applies to all cases, wherever the distance considered may be measured from. In applying the above formulæ the following points must be noticed :—

1. On substituting a numerical value for any of the symbols, the sign of the former must always be attached.

For example, if in formula (1), $u = 6$ and $v = -8$, then, on substitution, we get—

$$\frac{1}{-8} + \frac{1}{6} = \frac{1}{f} = \frac{2}{r}.$$

$$\therefore \frac{1}{24} = \frac{1}{f}.$$

$$\therefore f = 24 \text{ and } r = 48.$$

2. In applying a formula to determine one of the involved distances, the others being known, no sign must be given to the unknown distance. Thus, in the above example, no sign is at first given to f ; but the result, when worked out, shows it to be positive—that is, the mirror is concave.

3. When distances are measured from the pole of the mirror [formulae 1, 3 (a), and 3 (c)], the radius of curvature and focal length are **positive** for a **concave** mirror, and **negative** for a **convex** mirror. This is in accordance with the sign convention given above, and needs special notice only as a reminder.

4. Always draw a fairly accurate figure representing given conditions. This prevents mistakes as to sign.

Formulae 1 and 3 (*a*) are the most important.

Formulae 3 (*a*) and 3 (*b*) should be learnt *in words* (Art. 34); 3 (*c*) is not important, but is sometimes very convenient. The different forms of formula 3 may be remembered by noticing that "image" and "*v*" are associated, as are also "object" and "*u*."

5. Sign need not be considered in connection with formula 3 if the ratios be learnt *in words*. But if learnt as formulæ involving *u*, *v*, *f*, *c*, and *c'*, then the signs must be considered, just as in any other case, and the following interpretations of the results will be found useful:—

3 (*a*). Ratio *positive* indicates that image is *real*,
 " *negative* " " " " *virtual*,
 because, if *v* and *u* are of opposite sign, then the image must be behind the mirror, and therefore virtual (Art. 34).

3 (*b*). Ratio *positive* indicates that image is *erect*,
 " *negative* " " " " *inverted*,
 because, in the first case, the object and image must be on the same side of C, and the latter is therefore *erect*; and, in the second case, they are on opposite sides of C, and therefore the image is *inverted* (Art. 34).

3 (*c*). Ratio *positive* indicates that image is *real*,
 " *negative* " " " " *virtual*.
 Cp. 3 (*a*) and Art. 34, I. 3 and II. 1.

When the magnification is one of the data of a problem, attention must be paid to this point. See Ex. III. 4.

EXAMPLES III.

1. An object is placed 15 cm. in front of a concave mirror of 30 cm. focal length. Find the position of the image and the ratio of its size to that of the object.

Here we have given us—

$$u = 15; f = 30.$$

Hence, substituting in—

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f},$$

we have—

$$\frac{1}{v} + \frac{1}{15} = \frac{1}{30}. \quad \therefore \frac{1}{v} = -\frac{1}{30}.$$

$$\therefore v = -30.$$

That is, the image is 30 cm. *behind* the mirror, and is therefore

virtual. Also, image and object are on the same side of C; therefore image is *erect*.

Also—

$$\frac{\text{Image}}{\text{Object}} = \frac{v}{u} = \frac{-30}{15} = -2.$$

That is, image is *virtual*, and twice the size of the object.

2. A pencil of rays, converging to a point 20 cm. behind a mirror, is brought to focus, by reflexion from its surface, at a point 10 cm. in front of the mirror. Determine whether the mirror is convex or concave, and find its radius of curvature.

$$\text{Here } u = -20, v = 10,$$

$$\text{and } \frac{1}{v} + \frac{1}{u} = \frac{2}{r}.$$

$$\therefore \frac{1}{10} - \frac{1}{20} = \frac{2}{r}. \quad \therefore \frac{2}{r} = \frac{1}{20},$$

$$\text{or } r = 40 \text{ and } f = 20.$$

That is, the mirror is concave, and its radius of curvature is 40 cm.

3. An object, 3 cm. in length, is placed 20 cm. in front of a convex mirror of 12 cm. focal length. Find the nature and position of the image.

$$\text{Here } u = 20, f = -12$$

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}.$$

$$\therefore \frac{1}{v} + \frac{1}{20} = -\frac{1}{12}.$$

$$\therefore \frac{1}{v} = -\frac{1}{20} - \frac{1}{12} = -\frac{2}{15}.$$

$$\therefore \frac{1}{v} = -\frac{2}{15}.$$

$$\therefore v = -7.5.$$

That is, the image is 7.5 cm. *behind* the mirror, and is therefore *virtual*.

Also—

$$\frac{\text{Image}}{\text{Object}} = \frac{v}{u} = -\frac{7.5}{20}.$$

That is, image is *virtual*; and disregarding sign, we have—

$$\frac{\text{Length of image}}{3 \text{ cm.}} = \frac{3}{8}.$$

$$\therefore \text{Length of image} = \frac{9}{8} = 1.125 \text{ cm.}$$

4. A small object on the axis of a concave mirror, at a distance of 10 inches from it, gives an image which is three times its own length. Find the focal length of the mirror.

Here, applying 3 (c), we have—

$$\frac{\text{Image}}{\text{Object}} = \frac{f}{u-f}.$$

That is, since image may be *real* or *virtual*, we have—

$$\pm 4 = \frac{f}{10-f}.$$

$\therefore f = 8$ (for a *real* image),
or $13\frac{1}{3}$ (for a *virtual* image).

7. Given a concave mirror whose focal length is 12 inches, where would you place a candle flame in order that the image of it, formed by the mirror, may be (1) *real*, (2) *virtual*.

8. A concave spherical mirror is so placed that a candle flame is situated on its principal axis at a distance of 18 inches from its surface. An inverted image, three times as long as the candle flame itself, is seen sharply defined on the wall. What is the focal length of the mirror?

9. Prove that if an object is placed at a distance of $3f$ in front of a concave mirror (of focal length f), then the image is one-half the size of the object.

10. A small object on the axis of a concave mirror, at a distance of 16 inches from it, produces a *real* image which is three times its own size. Find the focal length of the mirror.

11. A small object 0.1 inch long is placed at a distance of 3 feet from a convex mirror of 12 inches focal length. What is the length of the image and its distance from the mirror?

13. A penny is held 8 inches in front of a convex mirror of 1 foot radius. Where will its image be, and what will be its diameter compared with that of the penny?

17. An object is held in front of a convex mirror, at a distance equal to the focal length of the mirror. Determine the size, nature, and position of the image.

19. An image produced by a convex mirror of focal length f is $1/r$ th the size of the object. Show that the distance of the object from the mirror is $(r-1)f$.

EXAMINATION QUESTIONS.

QUESTIONS SET AT LONDON UNIVERSITY EXAMINATIONS.

Matriculation.

1. A plane mirror revolves about an axis. Explain a method of ascertaining experimentally whether or not the axis is perpendicular to the surface of the mirror. *June, 1871.*

2. Two plane mirrors are inclined at an angle of 60° : trace the path of a pencil of rays proceeding from a luminous point between the mirrors to the eye, after undergoing one reflexion at the surface of each mirror. *Ibid.*

3. State the laws of the Reflexion of Light by plane-polished surfaces, and explain fully an accurate method of proving them by experiment. *Jan., 1872.*

4. If a small object on the principal axis of a concave mirror is gradually moved up to the mirror from a point at a considerable distance, show what will be the simultaneous changes in the position and size of the image. *Ibid.*

5. Apply the laws of Reflexion of Light to find the apparent position of a luminous point seen by reflexion in a plane mirror. *June, 1872.*

6. A ray of light is reflected successively by two plane mirrors, the plane of incidence being perpendicular to the line of intersection of the mirrors: prove that when the mirrors are at right angles to each other the final direction of the ray is parallel to its original direction. *Ibid.*

7. When a ray of light falls upon a rotating mirror, show that the reflected ray turns twice as fast as the mirror. *Jan., 1873.*

8. Enunciate completely (in two statements) the law of Reflexion of Light. Employ it to find the positions of the images of a bright point placed between two parallel plane mirrors. *June, 1873.*

9. A candle-flame is placed at a distance of three feet from a concave mirror formed of a portion of a sphere the diameter of which is three feet. Determine the nature and position of the image of the candle-flame produced by the mirror, and state whether it is erect or inverted. *June, 1874.*

10. Sketch a concave spherical mirror exhibiting a distant luminous object, and showing the position and nature of the image of this object given by the mirror. *June, 1875.*

11. Given a concave spherical mirror, how could you find its radius of curvature by optical means alone, and without resorting to geometrical operations?
Jan., 1876.

12. Rays of light from a bright gas flame pass through a small pinhole in a black screen, and are received on a sheet of ground glass. Describe by the help of a picture the image seen on the glass. What would be the effect of making the pinhole square instead of round?
Jan., 1877.

13. Completely enunciate in two statements the law of Reflexion of Light, and show how to find the chief focus of a concave spherical mirror.
Ibid.

14. Assuming the laws of the ordinary Reflexion of Light, find the position of the image of an object placed in front of a plane mirror. What are the limits of position of the object (the mirror being supposed fixed) so that an image of it may be formed by the mirror?
June, 1877.

15. A plane mirror, in the shape of a circle, revolves about a vertical diameter. A fixed horizontal ray of light falls upon its centre and is there reflected. Prove generally that if the mirror move through any angle the reflected ray will appear to have moved through double that angle.
Jan., 1878.

16. An object 6 inches long is placed symmetrically on the axis of a convex spherical mirror, and at a distance of 12 inches from it. The image formed is found to be 2 inches long. What is the focal length of the mirror?
Ibid.

17. Show how to find the position of the image of an arrow placed in front of a concave spherical mirror. Explain when it is an erect, and when an inverted image.
June, 1878.

18. Explain the formation of images by a concave cylindrical mirror. Find the relation between the distances of the two conjugate foci from the mirror. What is the position of the image of a point which is at the distance of the diameter from the reflecting surface of the cylinder?
Jan., 1879.

19. A small object is placed in front of a concave spherical mirror of 6 inches radius at a distance of four inches from the surface of the mirror. Where will its image be situated? will it be erect or inverted? and what will its dimensions be compared with those of the object? Where must the object be that the image may be of the same size?
Jan., 1880.

20. Explain the formation of images by means of a concave spherical mirror. How would you determine the focal length of such a mirror?
June, 1880.

21. Explain the formation of an image by a convex mirror. The radius of a convex mirror is 6 inches. If the linear dimensions of an object be twice those of its image, where must each be situated?
Jan., 1881.

22. State the laws of Reflexion of Light. Two mirrors are placed parallel to one another, and a luminous point is placed midway between them. Show how to draw accurately the path of a ray of light which, after undergoing 3 reflexions at one mirror and 4 at the other, enters an eye also placed midway between the mirrors, but at some distance from the source of light. *June, 1882.*

23. On a moonlight night when the surface of the sea is covered with small ripples, instead of an *image* of the moon being seen in the sea, a long band of light is observed on the surface of the sea extending towards the point which is vertically beneath the moon. Account for this phenomenon in accordance with the laws of reflexion, illustrating your explanation by a figure. *Ibid.*

24. A bright object is placed between two plane mirrors inclined at 45° . Draw a picture showing the path of a ray of light proceeding from the object and reaching the observer's eye after four reflexions. *June, 1886.*

25. Two mirrors are inclined to each other at right angles. Show that three images of an object, placed in the angle between the mirrors, are formed, and draw the pencil of rays by which the second image can be seen by an eye looking at one mirror. *June, 1887.*

26. State the optical law on which photometric measurements are based. A gas flame and a candle are eight feet apart, the former giving out nine times as much light as the latter. Show that there are two positions in which a screen may be placed so as to be equally illuminated by the two sources, and find these positions. *Jan., 1888.*

27. A candle flame is placed between two vertical plane mirrors inclined to each other at an angle of 45° . Draw a figure showing the path of a ray, which, after four reflexions, enters the eye of an observer at the same level as the candle. *Ibid.*

CHAPTER VI.

REFRACTION.

38. Refraction. We have seen that a ray of light travels in a straight line so long as its course lies in the same homogeneous medium, but when it passes from one medium into another it undergoes a change of direction at the surface of separation of the two media. This change of direction is called **refraction**. In illustration of this phenomenon the following simple experiments are frequently adduced:—

(1) When a piece of stick is partly immersed in water in an oblique position, it appears bent at the surface of the water (Fig. 38). This is due to the refraction of the rays coming from points on the stick below the surface of the

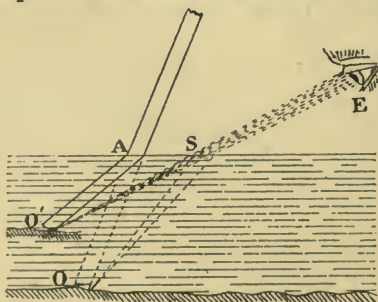


Fig. 38.

water. For example, rays coming from O (Fig. 38) are refracted at S in passing from the water to the air, and appear to come from O' . Similarly, other points between O and A appear, at E , to lie between O' and A , and thus the portion OA of the stick appears bent into the position $O'A$.

(2) If a coin be placed at the bottom of a vessel with opaque sides, in such a position as to be just out of the range of vision of an observer stationed a short distance off, it will be found that on pouring water into the vessel the coin soon becomes visible. Thus, if the coin be placed at S' (Fig. 3), it will be invisible to an eye at E , until, on pouring a sufficient quantity of water into the vessel, a small pencil of rays coming from S' and refracted at O , in passing from the water to the air, reaches the eye by the bent course $S' O A E$.

For similar reasons a pool of water appears shallower than it really is, and small air bubbles in solid glass objects appear nearer the surface than they actually are. (Art. 45.)

39. Angles of incidence and refraction. Let $A O$ (Fig. 39) represent a ray of light incident at O on the surface of separation of the media M and M' , and let $O B$ represent the refracted ray. Then, if $NO N'$ be the normal to the surface at O , the angle at $A O N$ is the *angle of incidence*, and $B O N'$ is the corresponding *angle of refraction*. The laws of refraction, as established by experiment, refer to the relative position and magnitude of these angles, and may be stated thus:—

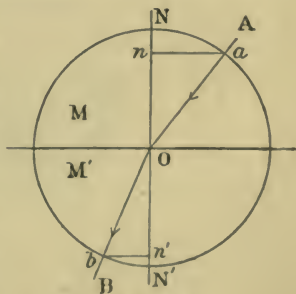


Fig. 39.

(i) *The angles of incidence and refraction lie in the same plane—that is, the incident and refracted rays, and the normal at the point of incidence, all lie in the same plane.*

(ii) *For the same two media, the ratio of the sines of the angles of incidence and refraction are constant, whatever may be the magnitude of these angles.*

(This law is generally known as the *law of sines*.) Without employing the term *sine* this law may be explained by a geometrical construction. With centre O

(Fig. 39) and any radius Oa describe the circle $a N b N'$, cutting OA and OB in a and b . From a and b drop perpendiculars an and bn' on the normal NN' . Then the law may be expressed by stating that, for the same media, the ratio $\frac{an}{bn'}$ is constant.

40. Experimental verification of the laws of refraction.

The laws stated in the preceding article may be roughly verified by means of the apparatus shown in Fig. 40. A cylindrical glass vessel, VV , is fixed, in a suitable stand, with its axis horizontal and its circular section vertical. A circular scale, divided into degrees, is fitted or engraved

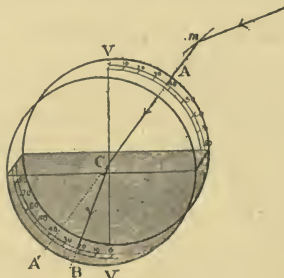


Fig. 40.

round its circumference. This vessel is half filled with water, holding a small quantity of freshly precipitated silver chloride in suspension, and the surface of the water is accurately adjusted on a level with the centre C of the circular scale. A pencil of parallel light is now reflected from a small mirror m , so as to be incident in the plane of the scale * on the surface of the water at C . The path of the refracted ray is rendered visible by the light dispersed by the particles of silver chloride, and is seen

* This adjustment should be made before the water is placed in VV . The scale is first adjusted in a vertical plane by altering the level of the apparatus until the plane of the scale is seen to coincide with a vertical thread hanging near; the position of the mirror m is then adjusted until the reflected beam, ACA' , cuts the scale at A and A' .

to be deviated from the direction of the incident ray immediately on entering the water. If the scale is accurately vertical, its plane will contain the normal to the surface of the water at the point of incidence, and the refracted pencil will be seen to lie in this plane, thus verifying the first law of refraction. Also, if the magnitudes of the angles of incidence and refraction be read off on the circular scale for several different values of each, it will be found that, in accordance with the law of sines, the ratio of the sine of the angle of incidence to the sine of the corresponding angle of refraction is constant.

41. Refractive indices. We have seen that, when a ray of light is refracted from one medium, a , into another, b , the ratio of the sine of the angle of incidence to the sine of the angle of refraction is constant. This ratio is the **relative index of refraction** from the medium a into the medium b . That is, if ${}_a\mu_b$ represent this index, and if ϕ and ϕ' denote respectively the angles of incidence and refraction, we may write—

$${}_a\mu_b = \frac{\sin \phi}{\sin \phi'}. \quad (1).$$

It has been established by experiment that the path of a ray of light is reversible—that is, if, in Fig. 39, BO be taken to represent the incident ray, then OA will be the path of the refracted ray. This fact is evidently expressed by writing—

$${}_b\mu_a = \frac{\sin \phi'}{\sin \phi}. \quad (2).$$

From (1) and (2) we have—

$${}_a\mu_b \cdot {}_b\mu_a = \frac{\sin \phi}{\sin \phi'} \cdot \frac{\sin \phi'}{\sin \phi} = 1.$$

That is—

$${}_a\mu_b = \frac{1}{{}_b\mu_a}.$$

Or—

$${}_b\mu_a = \frac{1}{{}_a\mu_b}. \quad (1).$$

This result may be stated in words by saying, that if ${}_a\mu_b$ denote the index of refraction from a to b , then $\frac{1}{{}_a\mu_b}$ denotes the index of refraction from b to a . For example, if the index of refraction from air to water be $\frac{4}{3}$, then the index of refraction from water to air is $\frac{3}{4}$.

When a ray of light is refracted from vacuum into any other medium, the index of refraction from vacuum into that medium is called the **absolute refractive index** or the **refractive index** of the medium.

If a ray of light pass from a given medium, through a layer of another medium bounded by parallel planes, into the medium in which it was originally travelling, it is known, from experiment, that the initial and final directions of the ray are parallel. This may either be taken as an experimental fact, or deduced from results already obtained from experimental data.

Thus, let AA and BB represent the parallel surfaces of separation of a layer of the medium b from a , and let $RN N'R'$ represent the path of a ray travelling from a through b into a again. Then, it is evident that the angles $R'N'n'$ and RNn are equal, for they have respectively the same relation to the equal angles $O'N'N$ and ONN' . Hence, $R'N'$ is parallel to RN , but

is not in the same straight line with it. It follows from this that when a ray of light passes from one medium through any number of layers of different media, having parallel surfaces of separation, back into the same medium, then the initial and final directions of the ray are parallel.

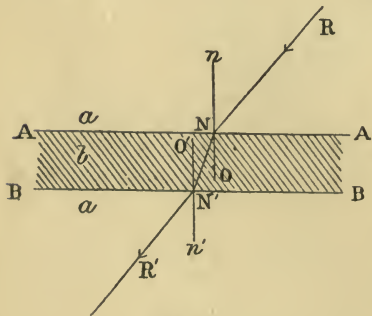


Fig. 41.

Consider the case for the three media a, b, c , shown in Fig. 42. Here—

$${}_a\mu_b = \frac{\sin \phi_1}{\sin \phi_2}$$

$${}_b\mu_c = \frac{\sin \phi_2}{\sin \phi_3}$$

$${}_c\mu_a = \frac{\sin \phi_3}{\sin \phi_1}$$

$$\therefore {}_a\mu_b \cdot {}_b\mu_c \cdot {}_c\mu_a = 1.$$

$$\therefore {}_a\mu_b \cdot {}_b\mu_c = \frac{1}{{}_c\mu_a} = {}_a\mu_c \text{ [by (1) above].}$$

That is—

$${}_a\mu_c = {}_a\mu_b \cdot {}_b\mu_c. \quad (2).^*$$

This is an important relation, and enables us to determine the relative index of refraction from a to c , given the indices

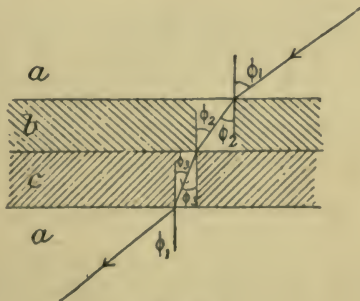


Fig. 42.

of refraction from a to b and from b to c . For example, if the index of refraction from air to glass, ${}_a\mu_g$, is $\frac{3}{2}$, and that

* This formula is easily remembered by noticing that a and c are the initial and final suffixes on *each* side. Compare the suffixes w, g , below.

from air to water, ${}_a\mu_w$, is $\frac{4}{3}$, then the index of refraction from water to glass, ${}_w\mu_g$, is given by—

$${}_w\mu_g = {}_w\mu_a \cdot {}_a\mu_g = \frac{1}{{}_a\mu_w} \cdot {}_a\mu_g = \frac{{}_a\mu_g}{{}_a\mu_w}.$$

That is—

$${}_w\mu_g = \frac{3}{2} \times \frac{3}{4} = \frac{9}{8}.$$

This formula also enables us to establish a relation between the **relative refractive index** for any two media and the **absolute refractive indices** of those media. Thus, if ${}_v\mu_a$ denote the absolute refractive index of the medium a , and ${}_v\mu_b$ that of b , then—

$${}_a\mu_b = {}_a\mu_v \cdot {}_v\mu_b,$$

or—

$${}_a\mu_b = \frac{{}_v\mu_b}{{}_v\mu_a}. \quad (3).$$

That is, *the relative index of refraction from a to b is the ratio of the absolute refractive index of b to the absolute refractive index of a .*

It should here be noticed that, as a general rule, a ray of light in passing from one medium into a denser one is bent towards the normal, while in passing into a rarer medium it is bent away from the normal. This is equivalent to stating, that if the medium b is denser than a , then—

$${}_a\mu_b > 1, \text{ and}$$

$$\therefore {}_b\mu_a = \frac{1}{{}_a\mu_b} < 1,$$

which expresses the case for refraction from b into the rarer medium a . Since all media are denser than vacuum, it follows that all absolute refractive indices are greater than unity.

So far we have considered the index of refraction merely as a geometrical relation, established by experiment, between the directions of the incident and refracted rays. When considered in connection with the undulatory theory of light, a definite physical meaning can, however, be attached to this constant. It can be shown that the index of refraction

tion from any medium a into another medium b is the ratio of the velocity of light in a to its velocity in b .

That is—

$${}_a\mu_b = \frac{V_a}{V_b},$$

where V_a denotes the velocity of light in a , and V_b denotes the velocity of light in b .

This ratio differs for waves of different wave length, being *greater* the *shorter* the wave length; and, as difference of wave length, in waves of light, corresponds to difference of *colour*, it follows that the value of the refractive index depends on the colour of the light which suffers refraction. The light having the *greatest wave length* and *lowest refractive index* is of a deep red colour, and that of the *shortest wave length* and *highest refractive index* is coloured *violet*. Between these two extremes the refractive index increases as the wave length decreases, and the colour of the light shades off from red through orange, yellow, green, blue, and indigo to violet.

Table of Refractive Indices.*
(Mean Values.)

Diamond	2.60	Hydrochloric acid	1.41
Flint glass	1.57	Alcohol	1.37
Rock-crystal	1.55	Ether	1.36
Rock-salt	1.54	Water	1.34
Canada balsam	1.53	Hydrogen	1.00014
Crown glass	1.52	Oxygen	1.00027
Plate glass	1.52	Air	1.00029
Iceland Spar	1.65	Nitrogen	1.00030
Alum.	1.45	Nitric oxide.	1.00031
Ice	1.31	Carbonic oxide	1.00034
Carbon disulphide	1.68	Ammonia	1.00039
Olive oil	1.47	Carbonic acid gas	1.00045
Oil of turpentine	1.47	Nitrous oxide	1.00050
Sulphuric acid	1.43	Chlorine	1.00078

* The student will find it convenient to remember the following approximate values :—

Refractive index for air and glass = $\frac{3}{2}$.

water = $\frac{4}{3}$.

No others need be learnt. "They" will be given if required in a question.

42. Critical angle. All possible values of an angle of incidence or refraction must evidently lie between 0° and 90° . Now, when a ray of light passes from a rarer into a denser medium, it is bent towards the normal—that is, the angle of refraction, ϕ' , is less than the angle of incidence, ϕ , and therefore, whatever be the value of ϕ , between 0° and 90° , that of ϕ' must also lie between 0° and 90° , and consequently refraction is always possible. But, if a ray of light pass from a denser into a rarer medium, it is bent away from the normal, and the angle of refraction, ϕ' , is greater than the angle of incidence, ϕ ; so that, when ϕ passes a certain limit at which ϕ' becomes equal to 90° , refraction is no longer possible, and the incident ray is **totally reflected** at the surface of the rarer medium.

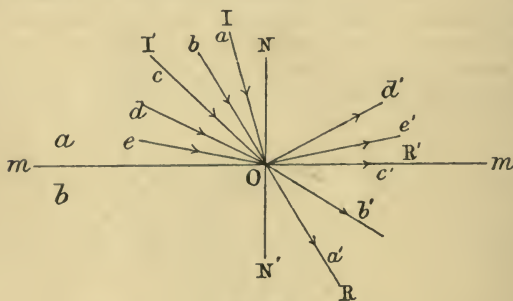


Fig. 43.

Let $m m$ (Fig. 43) represent the surface of separation of any two media a and b , of which b is the rarer, and let IO represent a ray incident, at O , at a small angle ION , and refracted along OR . As the angle of incidence increases and the incident ray takes the positions a, b, c , the angle of refraction also increases, and the refracted ray takes successively the corresponding positions a', b', c' . The angle of refraction being, however, greater than the angle of incidence, a position is reached at c where the angle of refraction $R'O N$ becomes equal to 90° , and the refracted ray, OR' , travels along the surface of separation of the media. The angle of incidence, $I'O N$, at which this takes place is the

critical angle for the media a and b . As the angle of incidence becomes greater than $I'ON$ the ray is no longer refracted into b , but is **totally reflected** from the surface nm in accordance with the ordinary laws of reflexion. Hence, as the incident ray passes through the positions d, e , it is reflected from nm along the corresponding paths d', e' .

Hence, *when refraction takes place from a denser into a rarer medium, the angle of incidence, which corresponds to an angle of refraction of 90° , is called the critical angle for the given media.* At this angle refraction ceases and total reflexion from the surface of separation of the media begins.

It should be noticed that for angles of incidence between 0° and the critical angle, only *a portion* of the light incident on the surface of the rarer medium is reflected at that surface, the remainder being refracted and dispersed (Art. 14); but, for angles of incidence greater than the critical angle, the incident light is almost totally reflected, no portion of it being refracted.

The value of the critical angle is readily determined for any media when the relative index of refraction for those media is given. Thus, let ${}_a\mu_b$ denote the index of refraction from a to b , then, in notation used above, if θ denote the critical angle, we have—

$${}_a\mu_b = \frac{\sin \phi}{\sin \phi'} = \frac{\sin \theta}{\sin 90} = \frac{\sin \theta}{1} = \sin \theta.$$

That is, the critical angle for refraction from a medium a into a rarer medium b , is the angle whose sine is the relative index of refraction from a to b , or—

$${}_a\theta_b = \sin^{-1} {}_a\mu_b. \quad (4).$$

This value of θ for air and water is about $48^\circ.30'$, and for air and glass it ranges from 38° to 41° according to the nature of the glass.

43. Total reflexion. As we have seen in the preceding article, total reflexion takes place when a ray of light, travelling in the denser of two media, is incident on the surface of separation at an angle greater than the critical angle of the media.

This phenomenon is readily exhibited by means of the apparatus shown in Fig. 40. The position of the mirror m is changed, and adjusted so as to reflect a beam of light upwards through the water into the air—for example, along the path BCA . As the angle of incidence is slowly increased the refracted ray gradually approaches the surface of the water, and finally, when the critical angle is passed, suffers reflexion at the surface of separation of the air and water, and is seen in the water as if reflected from a mirror coincident with this surface.

Simple illustrations of total reflexion are often met with. For example, if a glass vessel containing water be held above the level of the eye, and the surface of separation of the water from the air be looked at from below, it appears, when seen by total reflexion, as a brilliant reflecting surface.

Similarly, the edge of a crack in a pane of glass, seen obliquely, exhibits the same effect, as does also the surface of a glass tube held obliquely in a beaker of water when looked at through the sides of the beaker.

44. Deviation produced by refraction. Let AO (Fig. 44) represent the incident ray and OB the refracted ray, then the deviation produced by the refraction at O is expressed by—

$$D = A'OB = A'ON' - BON' \\ = AON - BON'.$$

That is—

$$D = \phi - \phi' \quad (5).$$

where ϕ denotes the angle of incidence and ϕ' the angle of refraction.

When the angle of incidence is zero, then the angle of refraction is also zero, and therefore no deviation is produced—that is, when a ray is incident along the normal to the surface of separation of two media it does not suffer deviation, but continues its course in the same straight line.

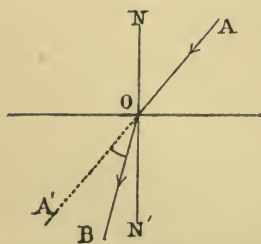


Fig. 44.

45. Refraction at a single plane surface. So far we have dealt only with the refraction of a single ray; we shall now consider the refraction of small pencils *directly** incident on the surface of separation of the media. Let mm (Fig. 45) represent the surface of separation of two media a and b , of which b is the denser, and let AB represent one of the extreme rays of a diverging pencil of light directly incident on mm along AN . AB is refracted at B along BC , while AN , being normal to mm , passes on along ND without suffering deviation. The focus of the refracted pencil will now be found at the point A' from which BC and ND apparently diverge. It thus appears that A' and A are conjugate foci, and that A' may be considered as the image of A formed by refraction.

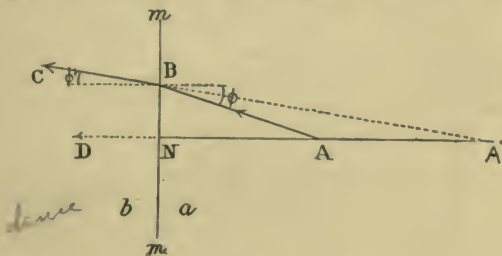


Fig. 45.

tion at the surface mm . It now remains to determine the relation between the distances of A and A' from that surface. Let ϕ and ϕ' denote the angles of incidence and refraction, and μ the refractive index† for the case considered. Then, since ϕ and ϕ' are respectively equal to the angles BAN and $BA'N$ (Euc. i. 29), we have—

$$\mu = \frac{\sin \phi}{\sin \phi'} = \frac{\sin BAN}{\sin BA'N} = \frac{BN}{BA} \cdot \frac{BA'}{BN} = \frac{BA'}{BA}.$$

But if BN is small—that is, if the incident pencil is small—

* A pencil of light is *directly* incident on a surface when the axis of the pencil is perpendicular to that surface.

† In what follows, μ always denotes the refractive index for refraction in the direction in which the light is supposed to be travelling.

then BA and BA' are approximately equal to NA and NA' , and we have—

$$\mu = \frac{NA'}{NA}.$$

Now, adopting the notation of Chapter V., and retaining the sign convention there explained, let NA be denoted by u and NA' by v . Then—

$$\mu = \frac{v}{u}.$$

$$\therefore v = \mu u. \quad (6).$$

That is, the distance of the image from the plane refracting surface is μ times that of the object.

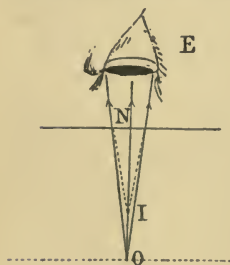


Fig. 46.

This explains why, on looking vertically downwards, the depth of a pond of water appears to be only three-fourths of what it really is. Let O (Fig. 46) represent an object at the bottom of the pond; then, after refraction at the surface of the water, the small direct pencil incident along ON appears to diverge from I —that is, the object O is seen at I . From relation (6) obtained above we get—

$$IN = \mu \cdot ON.$$

Now μ from water to air = $\frac{3}{4}$.

$$\therefore IN = \frac{3}{4} ON.$$

In exactly the same way the apparent thickness of a plate of glass, or other transparent medium, as seen by an eye looking along a normal to the surface of the plate, is less than its actual thickness. For if O (Fig. 46) represent an object close to the face of the plate remote from the eye, then its apparent position is at I —that is, IN is the apparent thickness of a plate of actual thickness ON . Hence, if t denote the thickness of the plate, its apparent thickness is given by μt , where μ is the index of refraction *from the medium into air*.

The result is true only in the case of small direct pencils.

Strictly, I is the conjugate focus of O only when the angle of the pencil diverging from O is infinitely small; as this angle increases, the focus I approaches nearer and nearer the surface (Fig. 47), until, when it is equal to twice the critical angle for the media, the point I coincides with N . If the angle of the pencil be greater than 2θ (where θ denotes the critical angle), then all the rays making angles greater than θ with the normals at the points of incidence are totally reflected, and do not emerge from the water. Hence, if O be seen, by an eye placed at E (Fig. 47) by means of the oblique pencil $O a b c d$, then its image is seen at O' , the apparent intersection of $a c$ and $b d$, and thus

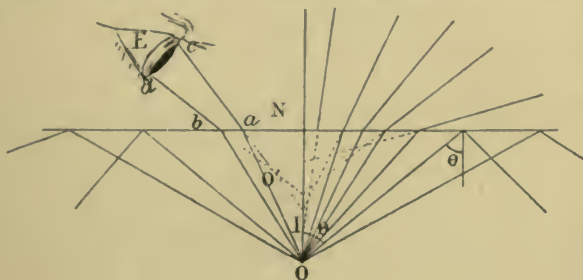


Fig. 47.

the apparent thickness of the medium becomes less and less as it is looked at more and more obliquely, and finally becomes zero when the direction of vision is parallel to its surface. This explains why the flat bottom of a vessel full of water appears slightly concave; the points vertically below the eye are seen by direct pencils, but the surrounding points by slightly oblique pencils, so that the water appears shallower as the range of vision travels outwards from the point vertically below the eye. If the eye be moved along parallel to the surface of the water, this appearance of concavity moves along with it, and thus an apparent wave motion is given to the bottom. For the same reason the depth of a pool of water appears to increase as we approach it and to diminish as we recede from it.

When a plate of glass or any transparent substance is interposed between the eye and a near object, the distance of the latter from the eye is apparently diminished. This is evidently due to the apparent diminution in the thickness of the plate; and, if t denote the actual thickness, then the apparent thickness, t' , is given by $t' = \mu t$, and the position of the object is apparently nearer the eye by a distance

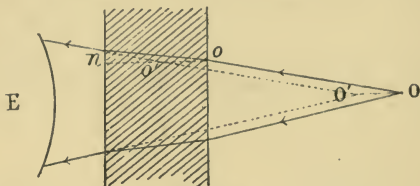


Fig. 48.

$(t - \mu t)$ or $t(1 - \mu)$, where μ denotes the index of refraction from the plate to air. Fig. 48 shows how this apparent change of position is effected. An object at O is seen at O' , the virtual focus of the refracted pencil which enters the eye at E . OO' represents the apparent change of position, and being equal to oo' is evidently equal to $(on - o'n)$; that is, if OO' be denoted by d , we have—

$$d = t(1 - \mu). \quad (7).$$

CALCULATIONS.

47. IN the preceding chapter several important relations have been established. For convenience of reference we shall here summarise the formulated expressions of these relations :—

$$(1) \quad {}_b\mu_a = \frac{1}{{}_a\mu_b}.$$

That is, the index of refraction from b to a is the reciprocal of that from a to b .

$$(2) \quad {}_a\mu_c = {}_a\mu_b \cdot {}_b\mu_c$$

$$(3) \quad {}_a\mu_b = \frac{v\mu_b}{v\mu_a}$$

$$(4) \quad {}_a\theta_b = \sin^{-1} {}_a\mu_b.$$

The relations (3) and (4) should be learnt in words.

$$(5) \quad D = (\phi - \phi')$$

$$(6) \quad v = \mu u$$

$$(7) \quad \begin{aligned} t' &= \mu t \\ d &= t(1 - \mu) \end{aligned}$$

In formulæ which involve v and u , distances are measured from the surface of separation of the media, and the usual sign convention (Art. 37) is adopted.

In all cases μ denotes the index of refraction in the direction in which the light is travelling.

EXAMPLES IV.

1. The absolute refractive indices of diamond and glass are respectively $\frac{4}{3}$ and $\frac{3}{2}$. Find the relative indices of refraction from glass to diamond, and from diamond to glass.

Here, if ${}_g\mu_d$ denote the relative index of refraction from glass to diamond, we have, from (3)—

$${}_g\mu_d = \frac{v\mu_d}{v\mu_g} = \frac{5}{2} \div \frac{3}{2} = \frac{5}{2} \times \frac{2}{3} = \frac{5}{3}$$

$$\therefore {}_g\mu_d = \frac{5}{3}, \text{ and by (1)}$$

$${}_d\mu_g = \frac{3}{5}.$$

2. Find the critical angle for water and glass, given that the index of refraction from air to glass is $\frac{4}{3}$, and that from air to water $\frac{3}{2}$.

Of the media, water and glass, glass is the denser, and by (1) and (2) we have—

$${}_g\mu_w = {}_g\mu_a \cdot {}_a\mu_w = \frac{4}{3} \cdot \frac{3}{2} = \frac{2}{3}.$$

Now, if ${}_g\theta_w$ denote the critical angle for glass and water, then—

$${}_g\theta_w = \sin^{-1} {}_g\mu_w = \sin^{-1} \frac{3}{4}.$$

That is, the critical angle for glass and water is an angle whose sine is $\frac{3}{4}$. Reference to a table of sines shows this to be $66^\circ 44'$.

3. A small air bubble in a piece of glass with a plane surface is 3 inches below that surface; find its apparent distance from an eye looking at it, along a normal to the surface, from a point 8 inches from the surface. (Index of refraction from air to glass $\frac{3}{2}$.)

Here, applying $t' = \mu t$ (7), and remembering that the light is supposed to be travelling from glass to air, and that therefore $\mu = \frac{2}{3}$, we have—

$$t' = \frac{2}{3} \times 3 = 2 \text{ inches.}$$

Therefore the apparent distance of the bubble from the eye = $8 + 2 = 10$ inches.

5. A piece of plate-glass, 5 inches thick (refractive index 1.6), is placed between the eye and an object. Find what alteration will take place in the apparent distance of the object from the eye.

6. Find the relative index of refraction from Canada balsam to air. (Refer to the table of refractive indices for data.)

7. The sine of the critical angle for two media is $\frac{1}{2}$. What is the index of refraction from the rarer to the denser of the two?

9. If a ray of light passes from one medium to a second, making the angle of incidence = 45° , and the angle of refraction equal to 30° , show that the refractive index for the media is $\sqrt{2}$.

10. The critical angle of a given medium is 60° . What is its refractive index?

Note.—When the *critical angle* or the *refractive index* of any medium is referred to, it must be understood that the other medium involved is vacuum.

11. A vessel, 6 inches deep, is filled with alcohol. What is the apparent depth of the liquid?

12. The refractive index of water is 1.33, and the velocity of light in air is 300,000,000 metres per second. Find the velocity of light in water.

CHAPTER VII.

REFRACTION THROUGH PRISMS AND LENSES.

48. In this chapter we shall not consider dispersion, and must therefore be understood to deal with the refraction of rays and pencils of light of *definite wave length*, and therefore of definite refractive index and *colour*. Such light is sometimes referred to as *monochromatic* or *homogeneous* light, and is conveniently obtained, of a yellow colour, from a flame coloured by the presence of a salt of sodium.

PRISMS.

49. **Prisms.** From an optical point of view, a **prism** is any portion of a medium lying between two plane faces inclined to each other at any angle. The line of intersection of these faces is known as the *edge* of the prism, and a section of the prism at any point in its length, perpendicular to this edge, is called a *principal section*. The *refracting angle* of the prism is the angle between its faces, as measured by the corresponding plane angle of the principal section. The prisms generally used for experiments are triangular prisms, in the geometrical sense of the term. The principal sections of such prisms are equilateral, isosceles, or scalene triangles, according to the purpose for which the prism is intended. When the section is equilateral the angle at each edge is equal to 60° , and thus there is no gain in having three edges; with an isosceles section there are two different angles available, and with a scalene section the angle at each edge is different, and thus the prism is equivalent to the three prisms considered in the optical sense.

50. Refraction through a prism. In dealing with refraction through a prism, we shall consider only the case where the plane of incidence and refraction is coincident with a principal section of the prism.

Let ABC (Fig. 50) represent the principal section of a prism, and BAC the refracting angle considered; then, if the material of the prism be of higher refractive power than the external medium, a ray RN incident on the face AC , at N , is bent towards the normal on entering the prism, and, taking the course NN' , is incident on the face AB at N' , where it is bent away from the normal, and leaves the prism by the path $N'R'$.

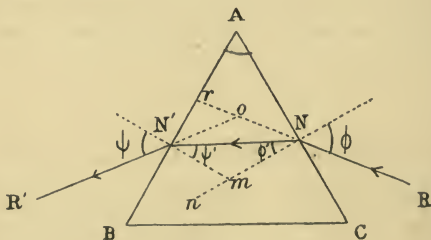


Fig. 50.

The ray RN is thus, after refraction through the prism, *deviated* from its original direction RN , and finally travels along $N'R'$. The deviation, as in Art. 44, is evidently measured by the angle $\rho o N'$; its *magnitude* is found to depend on the path of the ray through the prism, but its *direction* is always away from the refracting edge. There is one position for which this deviation is a minimum; when the prism is so placed that the incident and emergent rays make equal angles with the normals at their respective faces, then the deviation is a minimum, and the prism is said to be in the position of **minimum deviation**. This can be proved theoretically; but as the proof is beyond the scope of this work, we must, for the present, consider it as a fact established by experiment.

The minimum deviation produced by any prism depends on the angle of the prism and the refractive index of its material, relative to the external medium. We shall now proceed to establish an important relation between these quantities. In Fig. 50 let ϕ and ϕ' denote the angles of incidence and refraction at N, and ψ' and ψ the corresponding angles at N'.*

Then, if D denote the deviation produced, we have—

$$\begin{aligned} D &= r o N' = o N N' + o N' N. && \text{(Euc. i. 32).} \\ &= (\phi - \phi') + (\psi - \psi'). && [\text{Cp. Art. 44, Formula (5).}] \\ \therefore D &= \phi + \psi - (\phi' + \psi'). && (1). \end{aligned}$$

But, since the angle contained between any two lines is equal to that contained by lines perpendicular to them, we have, if A denote the angle of the prism—

$$\begin{aligned} n m N' &= B A C = A. \\ \text{But } n m N' &= \phi' + \psi'. && \text{(Euc. i. 32.)} \\ \therefore A &= (\phi' + \psi'). && (2). \end{aligned}$$

Substituting this value of $(\phi' + \psi')$ in (1), we get—

$$D = \phi + \psi - A. \quad (3).$$

Now, when the prism is in the position of minimum deviation, the ray passes *symmetrically* through the prism, and we must therefore have $\phi = \psi$ and $\phi' = \psi'$.

Therefore, from (3)—

$$D = 2\phi - A. \quad \therefore \phi = \frac{D + A}{2}. \quad (4).$$

And from (2)—

$$2\phi' = A. \quad \therefore \phi' = \frac{A}{2}. \quad (5).$$

But, if μ denote the refractive index of the material of the prism, relative to the external medium, then—

$$\mu = \frac{\sin \phi}{\sin \phi'}.$$

* At N' the angle $NN'm$ must be considered as the angle of incidence; but, for the sake of symmetry with ϕ' , it is here denoted by ψ' and not by ψ .

Therefore, substituting from (4) and (5), we have—

$$\mu = \frac{\sin \frac{1}{2} (D + A)}{\sin \frac{1}{2} A}. \quad (1).$$

This result, in connection with refraction through a prism in the position of minimum deviation, is of great practical importance.

When the angle of the prism is small, a very convenient expression for D may be obtained from the formula just established. Thus, we have—

$$\mu = \frac{\sin \frac{1}{2} (D + A)}{\sin \frac{1}{2} A}.$$

Now, if D and A be so small that the angles $\frac{1}{2} (D + A)$ and $\frac{1}{2} A$ may be considered as approximately equal to the sines of these angle, we have—

$$\mu = \frac{D + A}{A}, \text{ or, } D = (\mu - 1) A. \quad (2).$$

51. Conjugate foci in the case of refraction through a prism in the position of minimum deviation. It is a general law, that, when any quantity is passing through its maximum or minimum value, a small change in the variable concerned produces very little effect on the magnitude of the quantity itself. For example, the magnitude of the deviation produced by refraction through a prism depends upon the path of the rays, but when the prism is in the position of minimum deviation any small change in the path produces but little change in the magnitude of the deviation. Hence, for rays passing through a prism, by paths near to that of minimum deviation, the deviation which each undergoes is practically the same, and very nearly equal to the minimum value.

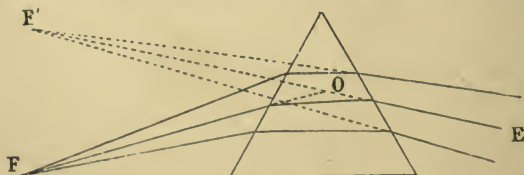


Fig. 51.

Hence, if a *small* pencil of rays coming from F (Fig. 51) be

incident on a prism at such an angle that the axis passes along the path of minimum deviation, then all the rays will be deviated to an approximately equal extent, and will therefore, on emergence, be inclined to one another at nearly the same angle as before incidence. Hence, if produced backwards, the rays of the pencil appear to come from a point F' such that $F'O = FO$. Similarly, if we imagine the path of the pencil to be reversed, we see that a convergent pencil having its focus at F' would, after refraction through the prism, converge to the point F .

F and F' are thus conjugate foci; and, if the pencil be incident near the refracting edge of a prism of small angle, placed in the position of minimum deviation for the axis of the pencil, we may neglect the thickness of the prism, and state that conjugate foci are on the same side of the prism and equidistant from its edge.

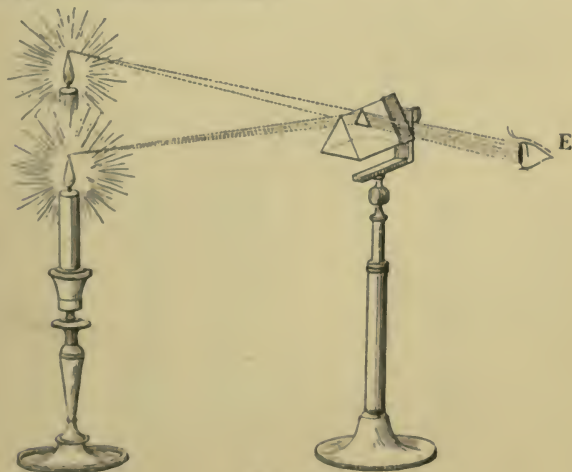


Fig. 52.

It follows, from what has been said above, that if an object be placed at F its image is seen at F' by an eye placed at E . This image is evidently virtual and displaced from the position of the object towards the edge of the prism (Fig. 52).

LENSES.

53. Lenses. A lens may be generally defined as a portion of a medium enclosed between two surfaces of definite geometrical form and having a common normal. Usually these surfaces are portions of spheres or plane surfaces, and the medium most generally employed is glass. Lenses of this form may be considered as solids of revolution. For example, if either of the sections shown in Figs. 54 and 55 be supposed to revolve round a central horizontal axis in the plane of the paper, the solids described by such revolution determine the form of the lens corresponding to each section.

It is usual to divide lenses into two classes:—

1. Convex lenses (Fig. 54). Of these there are three chief forms—

(a) Double **convex**.

(b) Plano-**convex**.

(c) Concavo-**convex** [*converging meniscus*].

The distinguishing characteristic of these lenses is that they are *thicker at the centre than at the edges*.

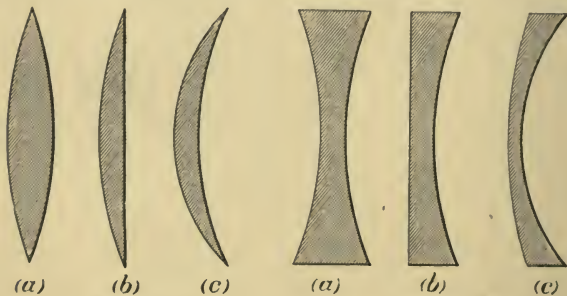


Fig. 54.

Fig. 55.

2. Concave lenses (Fig. 55). Corresponding to the three forms of convex lenses we have—

(a) Double **concave**.

(b) Plano-**concave**.

(c) Convexo-**concave** [*diverging meniscus*].

The distinguishing characteristic of this class is that the lenses are *thinner at the centre than at the edges*.

The action of any of these forms of lenses on a pencil of rays passing through them, depends on the refractive index of the medium of which they are made, relative to the surrounding medium. Usually we have to deal with glass lenses surrounded by air, that is, the medium of the lens is of higher refractive power than the surrounding medium. In this case, **convex** lenses cause the rays of a pencil to become more convergent, or less divergent after passing through them, and for this reason are sometimes called **converging** lenses. Similarly, **concave** lenses are called **diverging** lenses because the rays of a pencil are always more divergent or less convergent after refraction through them than before.*

This action of convex and concave lenses may be explained in the following way. The section of a double convex lens

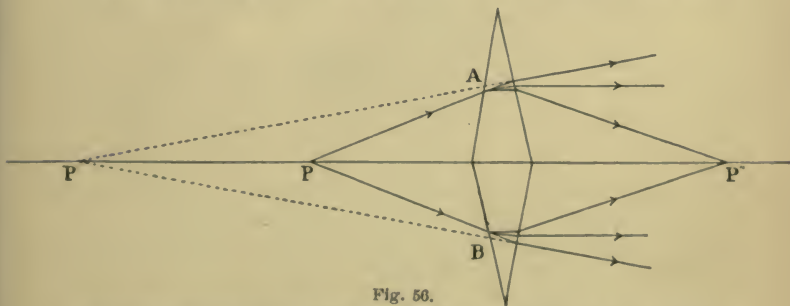


Fig. 56.

may be considered as a limiting form of that of two prisms placed base to base as in Fig. 56. Consider the rays PA and PB incident on the prisms at A and B. As explained in Art. 50, these rays are deviated away from the edges of the prisms on which they are incident, and are thus less divergent after refraction. The path of the rays PA and

* When the refractive index of the substance of the lens is less than that of the surrounding medium, then a *convex* lens acts as a *diverging* lens, and a *concave* lens as a *converging* lens.

PB, after passing through the lens, depends on the magnitude of the deviation produced; they may either diverge from P' , run parallel, or, if the deviation be sufficiently great, converge to a point, P'' .

Similarly, the section of a double concave lens may be considered as a limiting form of that of two prisms placed apex to apex, as in Fig. 57. In this case the rays PA and PB are refracted away from the edges of the prisms, that is, from the centre of the lens, and, after refraction, appear

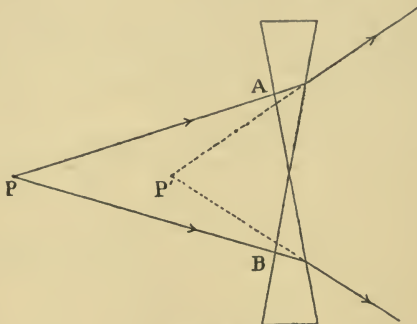


Fig. 57.

to diverge from the point P' ; the rays are thus more divergent after passing through the lens than before.

In the case of the prisms shown in Figs. 56 and 57, the positions of P' and P'' will depend on the positions of A and B, but in the case of a lens, owing to the curvature of the surface, all rays coming from P would, after refraction, pass through the same point. When this is accurately the case the curvatures of the surfaces of the lens are specially adapted to the existing conditions, and the lens is said to be *aplanatic*; but, for ordinary lenses, with spherical or plane surfaces, this is only approximately the case, and the defect resulting from this want of accuracy is known as *spherical aberration*.*

* When the surfaces of the lenses are only very small portions of spherical surfaces, spherical aberration is almost negligible, and the lens is, for all practical purposes, *aplanatic*.

A lens, being a solid of revolution, is symmetrical about its centre, and hence all sections passing through the axis of revolution are similar. It thus follows, that what has been explained above, for one section, is true for all similar sections, and consequently, if a *pencil* of light, diverging from P , be refracted through a lens, all the rays are symmetrically deviated, and, after refraction, pass through the same point.

54. Influence of curvature of surfaces of lens on deviation. Consider the refraction of the rays $PabP'$ and $PcdP'$ through the lens L (Fig. 58). It is evident from the figure

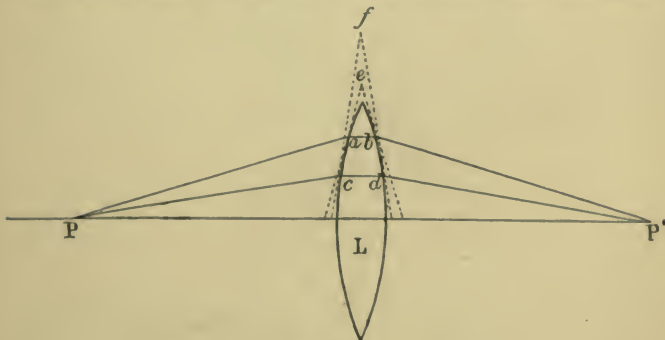


Fig. 58.

that, in order that the rays may pass through P' , the deviation of $PabP'$ must be greater than that of $PcdP'$. At a and b draw tangent planes to the surfaces of the lens meeting in e , and at c and d draw tangent planes meeting in f . Now the deviation in the case of the ray $PabP'$ is that due to the prism of refracting angle aeb , and the deviation for the ray $PcdP'$ is that due to a prism of angle cfa . But, when the angle of the prism is small, the deviation produced is approximately proportional to the angle of the prism (Art. 50). Therefore, in this case, the deviation for the ray $PabP'$ is greater than that for $PcdP'$, and thus it is possible for both rays to pass through P' .

55. Definitions. The *principal axis* of a lens coincides with its axis of revolution and, when the surfaces of the lens are spherical, passes through the centres of curvature of these surfaces. When one surface is plane, and the other spherical, the axis passes through the centre of curvature of the spherical surface and is normal to the plane surface. The **optical centre of a lens** is that point, on the principal axis, through which pass all rays having their paths parallel before and after refraction through the lens.

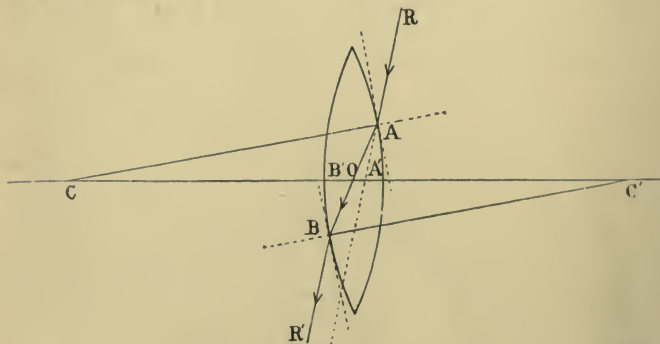


Fig. 59.

Let C and C' (Fig. 59) be the centres of the two spherical surfaces of a lens. Draw any radius CA, and through C' draw the radius C'B parallel to CA. Join AB cutting the principal axis CC' in O. Then O is the optical centre of the lens. For, if AB represent the path, *through the lens*, of the ray RABR', then, by construction, AB makes equal angles, at A and B, with the normals CA and C'B, and consequently the incident and emergent rays, RA and BR' also make equal angles with these normals, and are therefore parallel.* This is true for *any* two parallel radii, CA and CB, and hence O is the optical centre of the lens as defined above. To determine the position of O, we have, from the triangles AOC and BOC', that—

* The action of the lens is, under the conditions considered, exactly similar to that of a plate enclosed by the parallel tangent planes at A and B (cf. Art. 41).

$$\frac{CO}{C'O} = \frac{CA}{C'B} = \frac{CA'}{C'B'}$$

$$\therefore \frac{CA'}{C'B'} = \frac{CO}{C'O} = \frac{CA' - CO}{C'B' - C'O} = \frac{OA'}{OB'}$$

That is, the point O divides the thickness of the lens into segments proportional to the radii of curvature of the adjacent faces. In the case of double convex and double concave lenses the optical centre lies in the interior of the lens; in plano-convex and plano-concave lenses it is situated on the spherical surfaces, and in a converging or diverging meniscus it lies outside the lens on the same side as the surface of lesser radius of curvature.

Although the incident and emergent rays RA and BR' are parallel, they are not in the same straight line; but, if the thickness of the lens be small, the displacement produced is negligible, and it may be stated that all rays passing through the optical centre of the lens suffer no deviation, but continue their course in the same straight line.

Any line, other than the principal axis, passing through the optical centre is called a *secondary axis*.

When a parallel pencil of light is incident on a lens in a direction parallel to the principal axis of the lens, the rays, after refraction through the lens, converge to or diverge from a point on the principal axis. This point is the **principal focus** of the lens, and its distance from the optical centre of the lens, measured along the principal axis, is the **focal length** of the lens.

In the case of a convex lens, of any form, the parallel



Fig. 60.

pencil of rays is made to *converge* to a point F (Fig. 60) on the other side of the lens. A concave lens (Fig. 61) causes

the rays to *diverge* from a point F on the same side of the lens as the incident pencil. In both cases OF represents

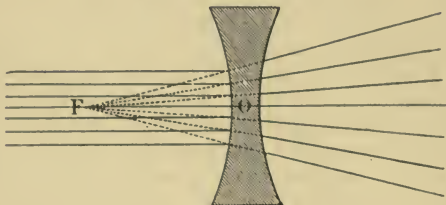


Fig. 61.

the focal length, and, applying the usual convention of sign, it is evident from Figs. 60 and 61 that, if distances be

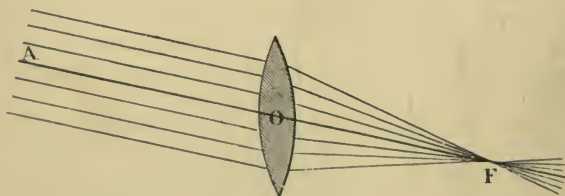


Fig. 62.

measured from O , the focal length of a *concave* lens is *positive*, and that of a *convex* lens *negative*.

If the pencil of parallel light is incident in a direction parallel to a secondary axis AF inclined at a *small* angle to the principal axis (Fig. 62) the focus of the refracted pencil is on the secondary axis at a point F such that OF is approximately equal to the focal length of the lens.

56. Path of a ray through a lens. Let C and C' (Fig. 63) denote the centres of curvature of the faces of the lens AB , and let the ray PA be incident on the surface of the lens at A . Join CA and produce it to N ; then CAN is the normal at A , and the ray PA is refracted

into the lens along AB making an angle BAC with the normal such that

$$\frac{\sin PAN}{\sin BAC} = \mu$$

where μ denotes the refractive index of the material of the lens relative to the surrounding medium. Similarly, at B , the ray is incident on the second face of the lens and is refracted along BP' in such a direction that

$$\frac{\sin P'BN'}{\sin C'BA} = \mu.$$

To determine, by this construction, the path of any given ray would be a very troublesome process; and it is therefore important to notice two particular cases in which the path is readily determined.

1. Any ray passing through the optical centre* of a lens continues its course in the same straight line (Art. 55).

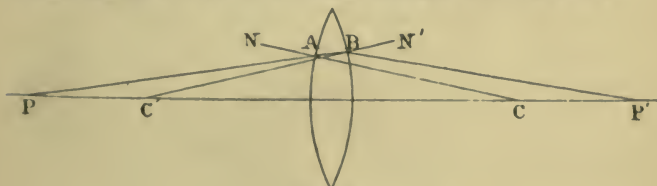


Fig. 63.

2. When the incident ray is parallel to the principal axis, the refracted ray passes through the principal focus (Art. 55).

57. Conjugate foci.† When rays of light, diverging

* For ordinary purposes the optical centre of a *thin* lens may be taken at any point in its thickness, on the principal axis.

† It should be noticed that if the conjugate foci are both *real* the image of an object placed at either focus is formed at the other; but if one of the foci is *virtual*, then the image of an object placed at that focus is not formed at the other, but rays converging to the *virtual* focus are refracted through the conjugate focus. That is, the optical relation between conjugate foci involves reversal of the direction of the light.

from a point P (Fig. 64) on the principal axis of a lens, are refracted through the lens, the focus of the refracted pencil is another point P', also on the principal axis. These points, P and P', are called *conjugate foci*.

When the point P is on any secondary axis *inclined at a small angle to the principal axis* the point P' is also on that secondary axis, but it is important to notice that secondary axes have not the same relation to lenses as they have to mirrors. In the case of mirrors, the secondary axes have exactly the same geometrical relation to the spherical reflecting surface as the principal axis, but for lenses this is not the case, and refraction along secondary axes involves several complications which we cannot now consider. When, however, the angle between a secondary axis and the principal axis is *small* the laws applicable to refraction along

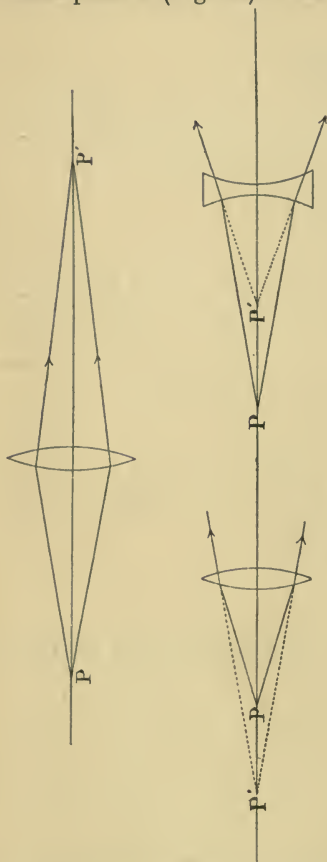


Fig. 64.

the principal axis may be applied with approximately correct results.

The relation between the distances of *conjugate foci* from the centre of the lens, and the focal length of the lens, will

be deduced in a later Article by a simple geometrical method.

We may here so far anticipate the result as to state that if u denote the distance of P from the lens, v the distance of P' , and f the focal length of the lens, then—

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}.$$

By application of this formula it is possible to determine the position of P' corresponding to any given position of P . For example, if u be infinite, that is, if the incident light be parallel, we have—

$$\frac{1}{v} - \frac{1}{\infty} = \frac{1}{f}.$$

$$\text{But } \frac{1}{\infty} = 0. \quad \therefore \frac{1}{v} = \frac{1}{f}, \text{ or } v = f.$$

This means that, if the incident light is parallel, then the focus of the refracted pencil is at the principal focus of the lens (Art. 55).

To trace the path of P' , as P travels, along the principal axis of a lens, from infinity up to the lens, the simplest method is to start from the definition of *principal focus* given in Art. 55, and apply the following rule:—*When an image is formed by refraction through a lens, any motion of the object along the principal axis produces a corresponding motion of the image in the same direction.*

Considering first a convex lens (Fig. 66), when P is at

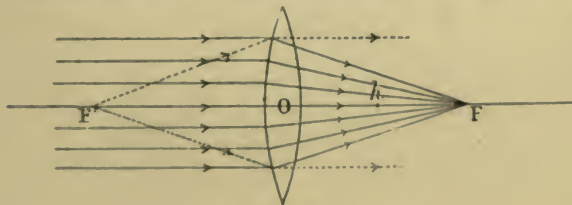


Fig. 66.

infinity, that is, when the incident light is parallel, then, by definition, P' is at F , the principal focus of the lens. As P travels from infinity up to F , P' travels, *in the same direction*,

from F to infinity, and the rays of the refracted pencil are now parallel. As P travels from F' to O , P' , after disappearing at infinity to the right of the lens, reappears from infinity to the left of the lens, and travels, *in the same direction as P* , up to O , where image and object coincide.

Similarly, in the case of a concave lens (Fig. 67), when

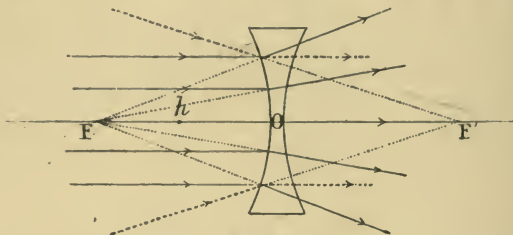


Fig. 67.

P is at infinity, P' is at F , that is, when the incident light is parallel, the rays of the refracted pencil diverge from the principal focus F . As P travels from infinity to O , P' travels, *in the same direction*, from F to O , where image and object coincide. From this it appears that *the image formed by a concave lens always lies between the principal focus and the lens*.

58. General construction for images formed by lenses.

Let AB (Figs. 68 and 69) represent an object placed on

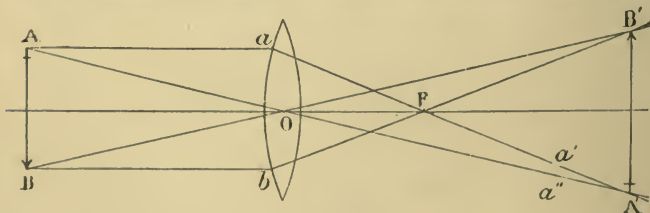


Fig. 68.

the principal axis of a lens. To determine the position of the image of the point A , it will be sufficient to determine

the point of intersection, after refraction through the lens, of any two rays originally diverging from A. We have seen, in Art. 56, that the path of a ray is readily determined when it is incident parallel to the principal axis, or passes through the centre of the lens. Let us consider, then, rays coming from A (Figs. 68 and 69) along both these paths. The ray Aa , incident parallel to the principal axis, is refracted along aa' in a direction passing through F, the principal focus of the lens. The ray AO passing through O, the centre of the lens, suffers no deviation, but continues its course along the straight line AOa'' . The two refracted rays aa' and Oa'' actually intersect (Fig. 68), or appear to intersect (Fig. 69) at A' , which is, therefore, the

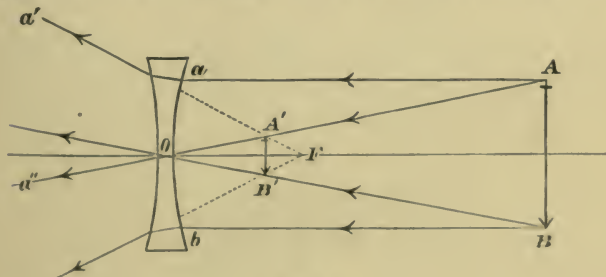


Fig. 69.

image of A. Similarly, the image of B is formed at B' , and the images of all points between A and B being assumed to lie between A' and B' , the complete image $A'B'$ is determined. When the rays really intersect, as in Fig. 68, the image formed is said to be **real**, but when they only apparently intersect, as in Fig. 69, the image is **virtual**. A real image is always formed on the side of the lens opposite to that on which the object is placed, and may be received on a screen, or seen by an eye * so placed as to receive the rays involved in its formation. A virtual image, having no real existence, cannot be said to be formed anywhere, but it is always

* The eye must be at the distance of distinct vision from the image.

seen on the same side as the object by an eye placed on the opposite side of the lens. A virtual image cannot be received on a screen.

59. Relative position of image and object. The formula,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f},$$

deduced for conjugate foci, evidently establishes a relation between the distances of the object and image from the centre of the lens; for, an image is an assemblage of foci, conjugate to corresponding points on the object. This relation may be proved geometrically in the following way.

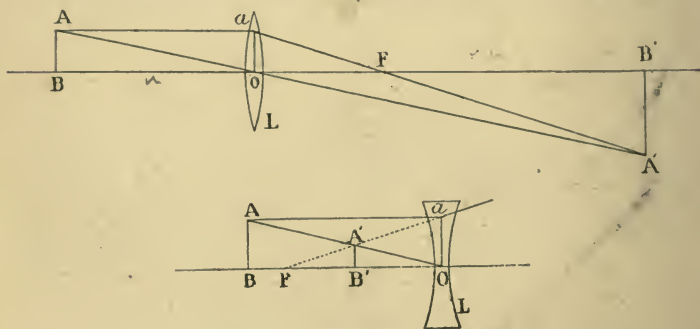


Fig. 70.

Let AB and $A'B'$ (Fig. 70) represent respectively an object and its image formed by the lens L . The construction for the image is identical with that explained above.

From the triangles $A'B'F$ and aOF we have—

$$\frac{A'B'}{aO} = \frac{B'F}{OF}, \quad (1) \text{ (Euc. vi. 4.)}$$

Similarly, from the triangles $A'B'O$ and ABO we have—

$$\frac{A'B'}{AB} = \frac{OB'}{OB}.$$

But—

$$AB = aO.$$

$$\therefore \frac{A'B'}{aO} = \frac{OB'}{OB}. \quad (2)$$

Therefore, from (1) and (2) we have—

$$\frac{B'F}{OF} = \frac{OB'}{OB}.$$

If now, OB be denoted by u ; OB' by v ; OF by f ; and the usual sign convention be observed, we get—

$$\frac{f-v}{f} = \frac{v}{u}.$$

$$\therefore uf - uv = vf.$$

$$\therefore uf - vf = uv.$$

Therefore, dividing by ufv , we get—

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}.$$

The variation of the position of the image with that of the object may be traced by the same method as that adopted in Art. 57 for conjugate foci.

The following particular cases should be noted :—

I.—Convex lenses.

1. If the distance of the object from the lens be greater than the focal length, the image is formed on the other side of the lens, at a distance also greater than the focal length, and is *real* and *inverted*. Fig. 68 illustrates the case; AB represents the object, and $A'B'$ the image.

2. If the distance of the object from the lens be less than the focal length, then the image is on the same side of the lens as the object, at a distance greater than the focal length, and is *virtual* and *erect*. Fig. 71 represents this case.

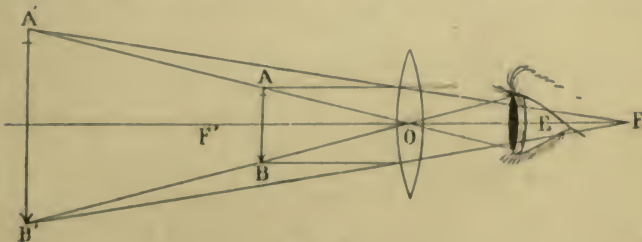


Fig. 71.

II.—Concave lenses.

Here we have only one general case. The image of an object, formed by a concave lens, is always on the same side of the lens as the object, between the principal focus and the lens; and its distance from the lens is always less than that of the object. Hence, the image formed by a concave lens is always *virtual, erect,** and *diminished*. Fig. 69 represents this case.

60. Relative size of image and object. In Fig. 70 let AB represent the object and $A'B'$ the image, then from the triangles AOB and $A'O'B'$ we have—

$$\frac{A'B'}{AB} = \frac{O'B'}{OB}.$$

$$\therefore \frac{\text{Image}}{\text{Object}} = \frac{v}{u}.$$

Also from formula,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f},$$

we have—

$$uf - vf = uv.$$

$$\therefore v(u + f) = uf.$$

$$\therefore \frac{v}{u} = \frac{f}{u + f}.$$

$$\therefore \frac{\text{Image}}{\text{Object}} = \frac{f}{u + f}.$$

From this relation the relative size of image and object may be determined without finding the position of the object. Considering the relation—

$$\frac{\text{Image}}{\text{Object}} = \frac{v}{u}$$

the following results are readily deduced—

1. If $v > u$, the image $>$ the object.
2. If $v = u$, the image $=$ the object.
3. If $v < u$, the image $<$ the object.

Applying those results to the general case I. 1 of Art. 59, we get three particular cases, according as the image is

* It should be noticed that *in the case of lenses a real image is always inverted, and a virtual image always erect.*

magnified, equal to the object, or diminished. In the second case, where the image is equal to the object, we have, for a *convex* lens, $v = -u$, and therefore in the formula—

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f},$$

we get—

$$\frac{2}{u} = \frac{1}{f} \text{ or } u = 2f.$$

That is, when the image formed by a convex lens is equal to the object, the distance of both image and object from the lens is equal to twice the focal length of the lens, and therefore the distance between the object and the image is equal to four times the focal length of the lens.

CALCULATIONS.

61. THE following relations, obtained in the preceding chapter, may again be noticed :—

1. Prisms.

$$\mu = \frac{\sin \frac{1}{2} (D + A)}{\sin \frac{1}{2} A}. \quad (1) \text{ (Art. 50.)}$$

$$D = (\mu - 1) A. \quad (2) \text{ (Art. 50.)}$$

This formula is approximately true only when A is small, and is rigorously true only when A is infinitely small. It should, therefore, not be used in calculations except when A is small; for example, less than 10° .

2. Lenses.

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}. \quad (4) \text{ (Art. 57.)}$$

$$\frac{\text{Image}}{\text{Object}} = \frac{v}{u}, \quad (5)$$

$$= \frac{f}{u + f}. \quad (5a) \text{ (Art. 60.)}$$

In the above formulæ all distances are measured from the centre of the lens, and the usual sign convention is adopted, that is, distances measured from the centre of the lens, in a direction **opposed** to the **incident light** are considered **positive**, and distances measured in the **same direction** as the **incident light** are considered **negative**. In accordance with this convention the **focal length** (f) of a **convex lens** will be **negative**, and that of a **concave lens** **positive**.

In applying the formulæ the rules given in Art. 37 must be attended to. Of these (1) and (2) are so important, and their neglect so often leads to mistakes, that we shall again deal with them in their relation to the formulæ here considered.

(1) On substituting in any formula a numerical value for any of the symbols, the sign of the former must always be attached. For example, take the formula—

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}.$$

If the image of an object, placed 20 cm. from a lens, be formed at a point 40 cm. *on the other side of the lens*, then, to find f , we have—

$$u = 20, v = -40;$$

and

$$\therefore \frac{1}{-40} - \frac{1}{20} = \frac{1}{f};$$

$$\therefore -\frac{3}{40} = \frac{1}{f};$$

and

$$f = -\frac{40}{3} = -13\frac{1}{3} \text{ cm};$$

that is, the lens is **convex**, and its focal length is $13\frac{1}{3}$ cm.

(2) In applying a formula to determine one of the involved distances, *no sign must be given to the unknown distance*. Thus, in the example given above, no sign is at first given to f , but the result, when worked out, shows it to be negative.

In applying formulæ (5) and (5a), which express the relative size of image and object, the question of sign should be carefully attended to, for the interpretation of the result is simple and important. In these formulæ, a *positive* result indicates that the image is *virtual* and *erect*; for the image and object are then on the same side of the lens. Similarly, a *negative* result indicates that the image is *real* and *inverted*, the image and object being then on opposite sides of the lens. (See footnote, Art. 59.) *When the magnification is one of the data of a question, this point must be attended to.*

EXAMPLES V.

1. The refracting angle of a prism is 60° , and the minimum deviation produced in a pencil of monochromatic light is 40° . Find the refractive index of the prism, given that $\sin 50^\circ = .766$.

Here, applying—

$$\mu = \frac{\sin \frac{1}{2}(A + D)}{\sin \frac{1}{2}A},$$

we get—

$$\mu = \frac{\sin \frac{1}{2}(60 + 40)}{\sin \frac{1}{2}(60)} = \frac{\sin 50}{\sin 30}.$$

$$\therefore \mu = \frac{.766}{\frac{1}{2}} = 1.532.$$

3. An object is placed 12 inches from a convex lens of 8 inches focal length. Find the position and nature of the image.

Here, in formula (4),

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f},$$

we have—

$$u = 12 \text{ inches}, f = -8 \text{ inches (convex lens)},$$

and v is required—

$$\begin{aligned}\therefore \frac{1}{v} - \frac{1}{12} &= \frac{1}{-8} \\ \therefore \frac{1}{v} &= -\frac{1}{8} + \frac{1}{12} = -\frac{1}{24} \\ \therefore v &= -24 \text{ inches;} \end{aligned}$$

that is, the image is 24 inches on the other side of the lens.

Again, applying (5), we have—

$$\frac{\text{Image}}{\text{Object}} = \frac{v}{u} = \frac{-24}{12} = -2;$$

that is, the image is twice the size of the object, and is *real* and *inverted*.

4. An object, 3 cm. long, is placed 10 cm. from a concave lens of 20 cm. focal length. Find the size and nature of the image. Here, from (5a) we get—

$$\begin{aligned}\frac{\text{Image}}{\text{Object}} &= \frac{f}{u + f} = \frac{20}{10 + 20} = \frac{2}{3} \\ \therefore \frac{\text{Length of image}}{3 \text{ cm.}} &= \frac{2}{3} \end{aligned}$$

\therefore Length of image = 2 cm., and the image is *virtual* and *erect*.

A more usual, but less direct method of working this question is, first to determine v , and then to determine the size and nature of the image from formula (5).

5. A concave lens whose focal length is 12 inches is placed on the axis of a concave mirror of 12 inches radius at a distance of 6 inches from the mirror. An object is so placed that light from it passes through the lens, is reflected from the mirror, again passes through the lens, and forms an inverted image coincident with the object itself. Where must the object be placed? *Matric., June, 1883.*

[In problems such as this, where by reflexion and refraction the image of the object is made to coincide with the object itself, the solution is easy if we remember that rays diverging from a point in the object, *on the principal axis*, return to the same point, and therefore travel to and fro by the same paths. But, if a ray, after reflexion at a mirror, return along its incident path, it follows that it must be travelling *along a normal to the mirror*.]

In this case we know that, after the first refraction through the lens, the rays of the refracted pencil—originally diverging from a point in the object on the principal axis—are normal to the mirror, and therefore diverge from its centre of curvature. To find the position of the object we have therefore only to find a point on the principal axis such, that rays diverging from this point appear, after refraction through the lens, to diverge from the centre of curvature of the mirror. ~

Hence, in the formula, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$, we have—

$v = 6$ inches, $f = 12$ inches, and u is unknown.

$$\therefore \frac{1}{6} - \frac{1}{u} = \frac{1}{12}.$$

$$\therefore \frac{1}{u} = \frac{1}{6} - \frac{1}{12} = \frac{1}{12}.$$

$$\therefore u = 12 \text{ inches.}$$

That is, the object must be placed 12 inches from the lens on the side remote from the mirror.

7. Show that if the angle of a prism be greater than twice the critical angle for the medium of which it is composed, no ray can pass through it.

8. The angle of a prism is 60° , and the refractive index of its material $\sqrt{2}$. Show that the minimum deviation is 30° .

9. A glass prism of refracting angle 5° is immersed in water; find the approximate deviation produced in a ray of light for which the absolute refractive indices of glass and water are respectively $\frac{3}{2}$ and $\frac{4}{3}$.

10. The minimum deviation produced by a hollow prism, filled with a certain liquid, is 30° ; if the refracting angle of the prism is 60° , what is the index of refraction of the liquid?

11. Show that when a ray of light is refracted through a prism, in the position of minimum deviation, the course of the ray in the prism is perpendicular to the line bisecting the angle of the prism.

15. A gas flame is at a distance of 6 ft. from a wall. Where must a convex lens, of 1 ft. focal length, be placed in order to give a distinct image of the flame on the wall? Explain your result.

16. An object, 1 inch long, is placed at a distance of 1 ft. from a convex lens of 10 inches focal length; find the nature and size of the image.

17. If an object, 10 cm. from a convex lens, has its image magnified 4 times, what is the focal length of the lens?

18. An object is at a distance of 3 inches from a convex lens of 10 inches focal length. Find the nature and position of the image.

19. An object is placed 6 inches from a lens, and an image, 3 times as large, is seen on the same side of the lens as the object. Find the focal length of the lens.

23. A convex lens of 6 inches focal length is used to read the graduations of a scale, and is placed so as to magnify them 3 times; show how to find at what distance from the scale it is held, the eye being close up to the lens.

24. The image formed by a convex lens is n times the size of the object. Show that the distance of the object from the lens is $\frac{n+1}{n}f$.

25. A candle flame is placed 6 inches from a plane mirror, and a convex lens, of 3 inches focal length, is placed between the candle and the mirror, and 2 inches from the latter. Find the position of the image.

26. A candle flame is placed 20 cm. from a plane mirror. Find where a convex lens of 5 cm. focal length must be placed in order that the image of the flame may coincide with the flame itself.

28. Show (by application of formula 5a above) that the image formed by a concave lens is always less than the object.

EXAMINATION QUESTIONS.

QUESTIONS SET AT LONDON UNIVERSITY EXAMINATIONS.

Matriculation.

1. Explain, in non-mathematical language, what is meant by the "Index of Refraction" of a transparent medium.

Also describe the phenomenon known as "total reflexion," and show how its occurrence in a given medium is connected with the index of refraction of that medium. *June, 1871.*

2. State the laws of the Refraction of Light by such substances as Water or Glass, and describe and explain experiments by which they can be demonstrated. *Jan., 1872.*

3. If a candle is placed at a distance of 6 feet from a wall, and a distinct image of the flame is produced upon the wall by a lens held at 1 foot from the candle, show that a distinct image will also be produced when the lens is at 5 feet from the candle, and compare the sizes of the two images. *June, 1872.*

4. A beam of light, on passing obliquely from air into water, is bent away from the surface of the water; and a straight stick, with one end immersed obliquely in water, appears to be bent towards the surface of the water. Show that these are illustrations of the same law of Refraction. *Jan., 1873.*

5. Explain how to draw a figure to represent the formation of a real magnified image of a small object by a lens. If a real image five times as high as the object is to be thrown on a screen at a distance of 36 inches from the object, show what must be the focal length of the lens employed. *Ibid.*

6. A beam of light issues from a given bright point 3 feet above the surface of still water, and falling obliquely on the surface, is divided into two parts, one of which is reflected and the other refracted. Find the position of the point of incidence and its distance from the bright point, so that the reflected and refracted beams may be at right angles to each other.

[The index of refraction from air to water is $\frac{4}{3}$.] *June, 1873.*

7. An object is moved from a considerable distance on the principal axis of a convex lens up to the lens. Find the corresponding changes in the position and size of the image. *Ibid.*

8. A bright point, 6 inches above the surface of still water, is reflected from the bottom of the vessel, which is 2 feet deep as well as from the surface of the water. Show how to find the positions of the images formed by the reflexions. ($\mu = \frac{4}{3}$) *Jan., 1874.*

9. Enunciate by aid of a sketch, and in two statements, the law of Refraction when light passes from a rarer to a denser medium. Also point out what is meant by the Critical Angle. *Ibid.*

10. A simple lens is used as a magnifier. Sketch the relative positions of the object (an arrow) and its image.

The same lens is used as in photography. Sketch the relative positions of the object and its image. *Ibid.*

11. Enunciate completely, by aid of a sketch, the law of Refraction. State what you mean by the Index of Refraction, and what by the Critical Angle.—The index of refraction from air to water is $\frac{4}{3}$: what is the sine of the critical angle in this case? *June, 1874.*

12. Prove that the apparent depth of a luminous object beneath a surface of water is only $\frac{3}{4}$ ths of its real depth.

[The index of refraction for water is $\frac{4}{3}$.] *June, 1875.*

13. When an object is to be photographed, an image of it is first obtained on a ground-glass screen, by means of the lens of the camera. Describe the nature and position of this image, and explain its formation. *Ibid.*

14. Explain "Angle of Reflexion," "Angle of Refraction," "Critical Angle." Is there any displacement of an object which is seen through a sheet of plate glass? Give reasons, aided by a diagram, for your reply. *Jan., 1876.*

15. Given the focal length of a convex lens, explain generally how it is possible to find the size of the image of the sun which such a lens will give. In what respect will this image be altered by diminishing the area of the lens without altering its curvature?

June, 1876.

16. A ray of light passes from air into glass, the refractive index of glass with regard to air being 1.5. Given the angle of incidence at the common surface, draw a diagram to show how the angle of refraction may be accurately determined. *Ibid.*

17. The chief focal length of a lens is 12 inches; how far must I place a luminous object from the lens in order to obtain an image twice as large every way as the object? *Jan., 1877.*

18. What is meant by a "refractive index" of a substance? Explain the fact that an aquarium tank appears to be much shallower (from front to back) than it really is; and point out in what way the difference between the apparent and true thickness is connected with the refractive index of the water in the tank. *June, 1877.*

19. Describe the two main categories under which lenses may be classified. A lens of water is enclosed in a rectangular envelope of glass. What kind of lens will the combination form?

June, 1877.

20. If the refractive index of a ray of light in passing from air to water be $\frac{4}{3}$, and in passing from air to glass $\frac{3}{2}$, find, by aid of a diagram, what it will be for the ray when passing from water to glass.

Jan., 1878.

21. Light proceeds from a point at the bottom of a lake. Make a careful drawing of the pencil of rays after emergence from the water, and find the geometrical focus of the pencil.

Jan., 1880.

22. What is the critical angle of a transparent medium? Describe what a fish would see on looking towards the surface of the water in directions differently inclined to the horizon, and illustrate your description by a diagram.

Ibid.

23. What is the focal length of a lens? A circle an inch in diameter, a convex lens whose focal length is 6 inches, and a second lens whose focal length is 10 inches are placed so as to have a common axis. The distance from the circle to the first lens is 10 inches, and from the first lens to the second 36 inches. What images of the circle will be formed, where will they be situated, and what will be their dimensions?

June, 1880.

24. How would you experimentally verify the laws of Refraction? What condition is necessary in order that a ray of light may be able to emerge from the plane surface of a refracting medium?

Jan., 1881.

25. What is the index of refraction of a transparent medium? What is the position of *minimum deviation* for a prism? Describe and explain the appearance presented when the image of a window is looked at through a prism with its edge vertical.

Jan., 1882.

26. Given the focal length of a lens, show how, by a geometrical construction, to find the position and magnitude of the image of an object whose distance from the lens is given.

An object whose length is 2 inches is placed 6 inches in front of a convex lens whose focal length is 4 inches. What is the length of the image?

Ibid.

27. Distinguish between a real image and a virtual image. Explain the action of a convex lens when used as a magnifying glass. Is the image seen by the eye real or virtual?

Ibid.

28. Show how to find the position and size of the virtual image of a given object, formed by a concave lens of known focal length. A concave lens whose focal length is 12 inches is placed on the axis of a concave mirror of 12 inches radius, at a distance of 6 inches

from the mirror. An object is so placed that light from it passes through the lens, is reflected from the mirror, again passes through the lens, and forms an inverted image coincident with the object itself. Where must the object be placed? *June, 1883.*

29. What is meant by the statement that the index of refraction of water is $\frac{4}{3}$? *Walking* by the side of a shallow stream of clear water of uniform depth, the gravelled bottom appeared to possess a wave-motion, the trough of the wave being always vertically beneath the observer. Explain this by means of a diagram. *Ibid.*

30. What is meant by the statement that the refractive index of water is 1.333? How is the critical angle for water found? An object is fixed one foot above the surface of still water; show how to find the apparent position of this object, as seen by an eye two feet vertically under it. *Jan., 1884.*

31. What is meant by saying that the refractive index of water with respect to air is $\frac{4}{3}$?

If the refractive index of water with respect to oil of turpentine be $\frac{9}{10}$, show how to find the refractive index of oil of turpentine with respect to air. *Jan., 1885.*

32. What is meant by the refractive index of a substance, and by total internal reflexion? Describe some experiment by which the phenomenon of total internal reflexion may be produced and observed. State also how the minimum angle of incidence at which total internal reflexion takes place may be determined. *June, 1886.*

33. An object 3 inches in height is placed at a distance of 6 feet from a lens, and a real image is formed at a distance of 3 feet from the lens. The object is then placed 1 foot from the lens. Where, and of what height will the image be? *June, 1887.*

CHAPTER VIII.

DISPERSION.

62. Homogeneous and compound light. We have seen that there is reason to believe that the physical cause of light is a species of transverse vibratory motion in the ether. When this motion is made up of a series of waves, all of the same wave-length, then the light is said to be *homogeneous* or *monochromatic*. It is, however, more generally the case that the wave motion is made up of an infinite number of waves of different wave-length. The light is then said to be *non-homogeneous* or *compound*.

Monochromatic light is of a definite colour, corresponding to its wave-length, and difference in wave-length is always indicated by a difference in colour. Compound light may also be of all shades of colour, or may be *white* or colourless, but its colour is no indication of its composition; two lights of almost identical colour may be made up of very different constituents, and may even exactly match the colour of any of the monochromatic lights. Thus the colour of light is not a reliable indication of its composition. In the case of white light however we can always state that it is compound, for all monochromatic lights are coloured, but, without experiment, we cannot state what may be the constituents of any given source of white light. Solar light and the other white lights with which we are most familiar, for example, gas-light, lamp-light, electric light, are very similar in composition, and include all possible shades of monochromatic light. The reason of this is evident when we remember that white light of this nature always results from incandescence, and that an incandescent or white-hot body has passed through all the phases of change of colour attendant on rise of temperature. It is therefore giving out light of all wave lengths, from

the dark red which first appeared when it began to get red-hot, to the violet which was added when it first appeared to be perfectly white.

63. Newton's Experiment. The light coming from the sun was first shown by Newton to be of a composite character. The experiment by which this fact was demonstrated is known as Newton's Experiment, and is worthy of special notice, both on account of the historical interest attached to it, and because of the great importance of the fact which it illustrates.

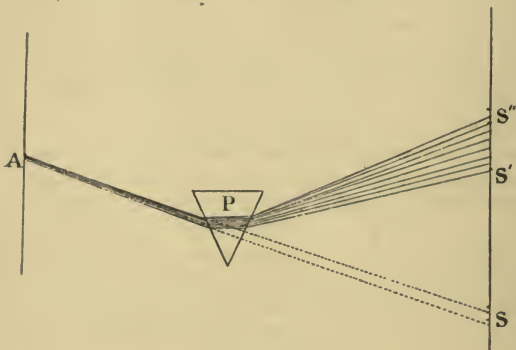


Fig. 72.

In its simplest form, Newton's Experiment may be performed in the following way. A beam of sunlight is admitted into a dark room through a small circular aperture, A, in a shutter or blind. This beam will be seen in the room as a small pencil of light diverging from A (Fig. 72), and, if allowed to fall on a vertical screen at S, it forms a small elliptical bright spot, which is a rough image of the sun. If now a prism, P, with its edge horizontal be placed, edge downwards, in the path of the beam, the latter will be deviated from its original course, and deflected upwards so as to form an image at S' S''. This image differs from that first formed at S in several

important particulars; the vertical diameter is much longer, and instead of appearing as a bright patch on the screen it is made up of several coloured bands, arranged horizontally. In fact the image is made up of several overlapping images, similar in shape to that originally seen at S, but each of a different colour. This shows that the beam of *white light* incident on the prism is, on refraction through the prism, separated into its different coloured constituents, each of which forms its own image on the screen, and thus the many-coloured compound image at S' S'' is formed. Such compound images are called *spectra*. When a *spectrum* is formed by decomposition of the solar light, as in the case we have just considered, it is called the *solar spectrum*, and is made up of the colours red, orange, yellow, green, blue, indigo and violet. Of these, the red rays are the least deviated, and therefore appear at S', the bottom of the image S' S''. The violet rays are the most deviated, and are therefore found at S'', the top of the same image. The intermediate rays are arranged in the order given above, from below upwards, between S' and S''.

It thus appears as if solar light were made up of only seven different constituents, corresponding to the seven colours given above. This is **not** the case; the number of constituents making up solar light are *infinite*, but, considered with reference to their action on the eye, they may be divided into seven sets, each of which corresponds to a definite *colour* sensation, and comprises an infinite number of rays, each corresponding to a certain *shade* of the colour which characterises the set to which it belongs.

64. Refrangibility. When the rays of a compound beam of light are refracted through a prism, as in Newton's experiment, each constituent suffers refraction to a different extent. This is sometimes expressed by saying that the constituents of the compound beam are of different *refrangibilities*; the most *refrangible* rays are those which undergo the greatest deviation, while the least *refrangible* are those which suffer least deviation. In the *solar spectrum* the red rays are the least refrangible, while the violet rays are the most re-

frangible. The intermediate rays increase in refrangibility as we pass from red through orange, yellow, green, blue and indigo to violet.

From what has been said above, it will be seen that difference in refrangibility corresponds to difference in wave-length. Light of high refrangibility is of short wave-length, and the corresponding index of refraction for any given medium is relatively high, while light of low refrangibility is of long wave-length, and the corresponding index of refraction for any given medium is relatively low.

65. Pure spectrum. The spectrum obtained by the method of Newton's Experiment is indistinct and badly

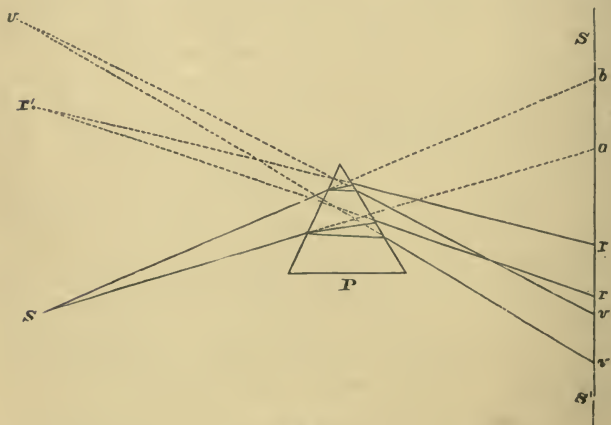


Fig. 73.

defined because of the overlapping of the images of which it is composed. Such a spectrum is said to be *impure*. To obtain a *pure* spectrum a very narrow slit must take the place of the aperture in the shutter, and some means must be adopted to obtain a spectrum made up of a series of adjacent but not overlapping images of this slit. In Fig. 73 let *s* denote the position of the slit. The pencil of rays,

diverging from s , forms a broad band ab on the screen SS' . If now the prism P be interposed, *in the position of minimum deviation*, with its edge parallel to the length of the slit, the rays of the pencil are deviated and dispersed in such a way that the red light appears to come from r' , a virtual image of the slit s , and forms a band rr on the screen. Similarly the violet light seems to come from v' and gives the violet band vv ; and so on, for each colour of the spectrum. It is evident that the spectrum thus obtained on the screen is composed of a series of overlapping bands, and is therefore not *pure*. If, however, a suitable lens L be placed, as shown in Fig. 74, so as to give, when P is removed, a dis-

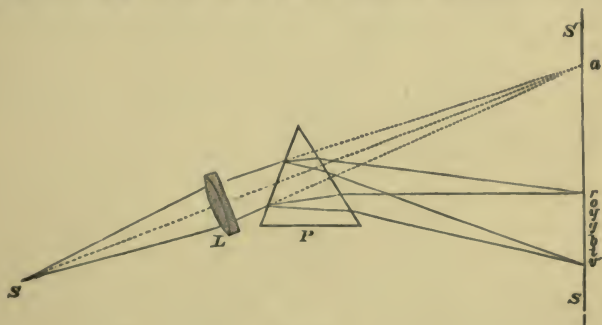


Fig. 74.

tinct image of the slit at a , and the prism, P , be then interposed, in the position of minimum deviation, the pencil of rays converging to a , will, after refraction through the prism, be dispersed and give rise to a series of pencils converging to the points r, o, y, g, b, i, v . Real images of the slit are thus formed at these points by light of each colour, and, as each image is narrow and distinct, like that at a , there is no overlapping and a *pure* spectrum is obtained. If the slit itself is not sufficiently narrow, the images may be broad enough to overlap and thus give an impure spectrum. Instead of placing the lens at L (Fig. 74), it may be placed on the other side of

the prism in such a position that real images of the virtual foci lying between r' and v' (Fig. 73) are formed on the screen.

It is thus evident that to obtain a pure spectrum we must have—

1. A very narrow slit.
2. The prism in the position of minimum deviation.
3. A lens, so placed as to form a *clearly defined* spectrum on the screen.

The second condition is of great importance, for it is only when the prism is in the position of minimum deviation that clearly defined images can be obtained. In practice it will be found convenient to illuminate the slit by means of a lamp, and instead of the large screen shown in the figure, to employ a smaller one, placed first at a to receive the direct image of s , and then at $r \dots v$ to receive the spectrum. The prism is placed in the position of minimum deviation by rotating it until the position of the spectrum is as near as possible to a . (Art. 81.)

It should here be noticed that the above arrangement of apparatus is necessary to obtain a *real*, pure spectrum, which can be received upon a screen. A *virtual*, pure spectrum can be seen by merely looking through a prism at a narrow slit. An eye placed near the prism, so as to receive the emergent pencil, sees a small, but very bright and pure spectrum at the virtual foci of the different constituents of the pencil entering the eye.

66. Dispersion. We have seen that when a beam of compound light is refracted through a prism, each constituent of the beam suffers deviation to a different degree. The light of shortest wave-length is deviated most, and that of longest wave-length least, and thus the different constituents of the incident beam are, as it were, separated, each travelling in a definite direction determined by the deviation it has experienced. This separation of the different constituents of a compound beam of light by refraction through a prism is called *dispersion*, and is measured, for any two rays of the refracted pencil, by the angle between these rays.

68. Dispersion in a lens. When a pencil of compound light is refracted through a lens, it suffers dispersion just as in refraction through a prism. Thus, if a diverging pencil of solar light, Pab (Fig. 77), be incident on the convex lens L , then the red rays, being the least refrangible, are brought to a focus at R , while the violet rays converge to a focus V nearer the lens. The orange, yellow, green, blue, and indigo rays converge to points intermediate between R and V , and thus, instead of the refracted rays all meeting in one focus, the rays of each colour converge to their own focus, and the image formed on a screen placed anywhere near V or R will be coloured at its edges. If the screen be placed anywhere near V , between the line vr and the lens, then the outer edge of the image will be red, but,

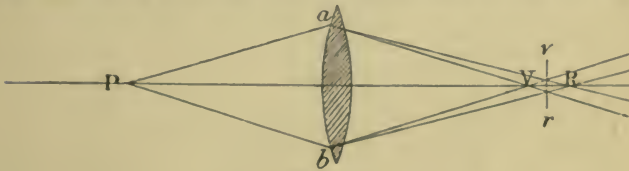


Fig. 77.

if placed beyond vr , then the outer edge shows violet. This fact is taken advantage of in focussing an image on a screen. The points V and R are very close together, and the best *definition* of the image is obtained when the screen is at vr . This adjustment is readily made by gradually changing the position of the screen until the colour showing at the outer edge of the image changes from red to violet. When this change of colour takes place the screen is in the position indicated by the line vr .

69. The Prismatic Spectrum. The *prismatic* spectrum is the spectrum obtained by the decomposition of white light on refraction through a prism.

All radiant waves are capable of refraction and dispersion, and thus, when a beam of white light is refracted through a prism, the emergent pencil is made up of a series of rays,

separated and arranged in order of *continuously* increasing refrangibility. Beginning at the less refrangible end of the spectrum determined by this emergent pencil, and travelling in the direction of increasing refrangibility, we pass a group of rays known as the *dark heat rays* which do not excite the sensation of sight. Then we come to another group, the *visible spectrum* ranging through the colours red, orange, yellow, green, blue, indigo and violet. This group of rays, in addition to possessing heating properties, has the peculiar property of exciting the optic nerve, and thus producing the sensation of sight. In this visible spectrum the intensity of the light is different in different parts, being a maximum in the yellow and gradually diminishing on both sides towards the red and violet. This is shown in Fig. 78, where the intensity of the light at any point on the

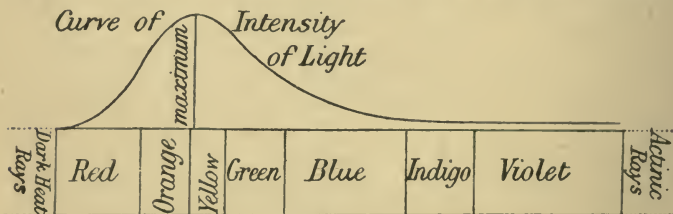


Fig. 78.

spectrum is directly proportional to the vertical height of the *intensity curve* above that point. Beyond the visible spectrum we come to the *dark chemical rays* or *actinic rays*. These extend for a considerable distance beyond the violet, and are characterised by their power of producing chemical action in a certain class of substances.

The complete spectrum is thus made up of the *dark heat rays*, the *visible spectrum* and the *actinic rays*. The only essential difference between these is that of wave-length, which *continuously* increases from the first to the last. All these rays have heating and actinic properties in different degrees and under different conditions, but only those of the visible spectrum have the power of exciting the sensation of vision. Fig. 78 gives a plan of the visible

spectrum and indicates how the intensity of the light varies along it.

70. Recomposition of white light. The colours of the spectrum may be recombined so as to produce white light. This may be done in several ways.

1. By combining two similar prisms in the way indicated in Fig. 76.

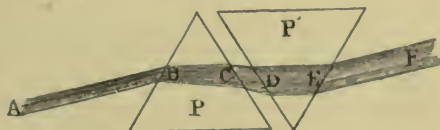


Fig. 76.

2. By allowing the dispersed beam, after emergence from the prism, to fall upon a lens. The image formed by the lens upon a suitably placed screen will be white, except at the edges where there will be a narrow fringe of colour.

3. If the dispersed beam be allowed to fall upon a concave mirror, the image formed by the reflected pencil will be white.

4. If a circular disc of cardboard be divided into seven sectors, painted in colours corresponding to the seven colours of the spectrum, and then rapidly rotated, it appears white. This result is due to the fact that visual impressions persist in the eye for a small fraction of a second, so that, if different impressions follow one another sufficiently rapidly, the final sensation is due to the *combined* effect of the individual impressions. Thus, in the case considered, the colour seen on looking at the rapidly rotating disc, is the combined effect of the different colours painted on the sectors.

CHAPTER IX.

VELOCITY OF LIGHT.

71. THE velocity with which light travels through any medium is inconceivably great, but varies with the nature of the medium. The velocity *in vacuo* is taken as the velocity of light, and the velocity in any other medium may then be determined, as explained in Art. 41, from the absolute refractive index of that medium. For, if V denote the velocity of light *in vacuo*, and V_m its velocity in any given medium, then—

$$\frac{V}{V_m} = \mu,$$

where μ denotes the absolute refractive index of the medium.

Hence, if we can determine the velocity of light in any medium, such as air, we can calculate its velocity in any other medium, or *in vacuo*.

The velocity of light has been determined in two general ways :—

- (1) From observations of celestial phenomena.
- (2) By terrestrial experiments.

The former of these gives, approximately, the velocity *in vacuo*, and the latter the velocity in air.

72. **Determination of the velocity of light from observations of celestial phenomena.** The first computation of the velocity of light by this method was made, in 1675, by **Roemer**, a Danish astronomer. He deduced his result from observations of the eclipses of Jupiter's nearest satellite. This satellite is eclipsed to us once during each revolution when it passes behind the planet into the shadow cast by the sun. The instant at which the eclipse should take place can be accurately calculated from dynamical con-

siderations. From a series of observations of the time at which the eclipse appeared to take place, Roemer noticed that the observed time was always *later* than the calculated time, and also that the difference between these two times varied with the relative position of the earth and Jupiter. From a careful analysis of observations it was found that the difference between the observed and calculated times of eclipse had a maximum and a minimum value corresponding to the positions of the earth at which its distances from Jupiter was a maximum or a minimum.

From this it is evident that the interval between the actual occurrence of the eclipse and the instant of its observation on the earth corresponds to the time taken by light in travelling from Jupiter, J (Fig. 79), to the earth, E,

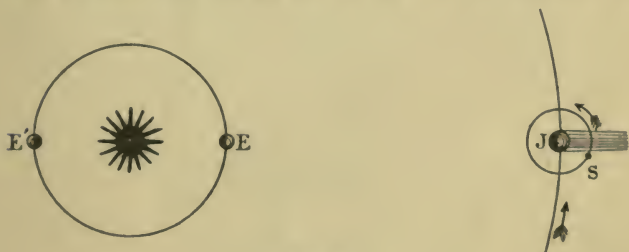


Fig. 79.

and that the *difference* between the maximum and minimum values corresponds to the time taken by light in travelling from E to E', which represent respectively the positions at which the earth and Jupiter are nearest together and farthest apart.

The distance E E' is the diameter of the earth's orbit, and is about 185,000,000 miles. The time of observation of the eclipse when the earth is at E' is about 16·5 minutes later than when the earth is at E. Hence we have—

$$\text{Velocity of light} = \frac{185,000,000}{16 \cdot 5 \times 60} \text{ miles per second.}$$

This gives a velocity of about 186,900 miles per second.

About fifty years after the time of Roemer, **Bradley**, the English astronomer, gave an explanation of the phenomenon of *astronomical aberration*, based on the fact that light travels through space with a definite velocity. This phenomenon is due to the fact that both the earth and light travel through space with definite velocities, and hence the direction in which light from a star reaches the earth will be in the direction of the velocity of the light *relative to the earth*. Thus, if A (Fig. 80) represent the position of the earth when light from a star, S,* starts from S', and if the velocities of the earth and of light be such that the former travels from A to B while the latter travels from S' to B, then the direction in which the star is seen from the earth is parallel to A B, and not to the true direction, A S''.

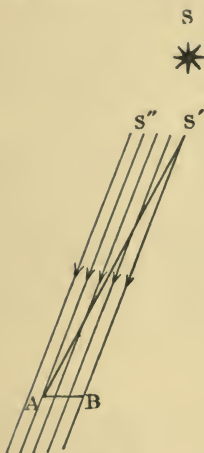


Fig. 80.

From our construction it is evident that—

$$\frac{AB}{BS'} = \frac{\text{Velocity of the earth}}{\text{Velocity of light}},$$

and, when the angle S'' A B is a right angle, that is, when the true direction of the star is at right angles to that in which the earth is moving in its orbit, we have—

$$\frac{AB}{BS'} = \tan B S' A = \tan S' A S'',$$

and the angle S' A S'' is called the *aberration* of the star.

* Many of the stars are at such great distances from the earth that, neglecting aberration, they are apparently seen in the same direction whatever the position of the earth in its orbit. The distance of the nearest fixed star is not less than 200,000 times the distance of the sun.

Hence, if V denote the velocity of light, and v the velocity of the earth in its orbit, we get—

$$\frac{v}{V} = \tan \theta,$$

where θ denotes the aberration of the star.

Of the quantities involved in this relation v and θ can be determined by astronomical observation, and V can then be calculated.

The value obtained for V by this method is about 185,000 miles per second.

73. Determination of the velocity of light by terrestrial experiment. Two distinct methods have been devised to determine the velocity of light by direct experiment.

1. Fizeau's method. The principle of this method is simple. Let S (Fig. 81) represent a source of light and M



Fig. 81.

a plane mirror. Now if a ray of light, SM , be incident normally on the mirror M , it will be reflected back along MS , and an observer behind S will see an image of S in the mirror. But, if a toothed wheel, having the teeth and spaces of equal width, be interposed at W in the position indicated in the figure, it may be rotated at such a rate that the light incident through any space will, after reflexion, be received on the back of the next tooth, and thus no image of S will be seen in the mirror. When this is the case it is evident that during the time taken by the

wheel to rotate through the angular width of one of the spaces, light travels from W to M and back again. Hence, to determine the velocity of light, from this experiment we have that—

$$v = \frac{2 \text{ W M}}{t},$$

where t denotes the time in which the wheel rotates through the angle subtended by one of the spaces, at the centre of the wheel.

2. Foucault's method. This method is somewhat complicated, both in theory and practice, and we shall therefore consider only the principle involved, without entering into the details of the apparatus actually employed. Let a (Fig. 83) represent a small bright object, m a plane mirror

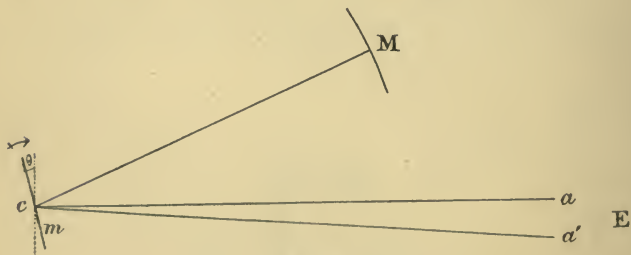


Fig. 83.

capable of rapid rotation, in the direction indicated by the arrow, round an axis perpendicular to the plane of the paper. M represents a concave spherical mirror placed so that *its centre of curvature is at c* , the axis of rotation of the plane mirror. For a certain position of m a ray of light, ac , starting from a is reflected from m to M , along the normal cM , and is thence reflected back along the same path to a . Hence, if an eye be placed at E an image of a will be seen in the direction ac .

If now the mirror m be made to revolve it will pass through the position just considered once in each revolution, and therefore an image of a will be seen, for an instant,

once in each revolution. When the revolutions become sufficiently rapid (about 30 per second) these quickly succeeding images persist on the retina, and blend into one permanent image, still apparently seen in the direction ac . When, however, the speed of rotation is greatly increased, the mirror, m , turns through an appreciable angle while the light is travelling from c to M and back again. For example, if the mirror turn through the angle θ (Fig. 83) while light travels from c to M and back to c , then the ray Mc will not be reflected along ca , but along ca' , and the eye at E sees the image of a in the direction $a'c$. Hence, if we can determine the angle $a'ca$ and the distance cM , we can calculate the velocity of light. For, by Art. 25, the angle $a'ca = 2\theta$, and light travels a distance $2cM$ during the time that the mirror revolves through an angle θ . If the mirror makes n revolutions per second, then the angular velocity per second is $2\pi n$, and the time in which the angle θ is described is given by—

$$t = \frac{\theta}{2\pi n} \text{ seconds.}$$

Therefore, if cM be denoted by l , the velocity of light is given by—

$$V = \frac{2l}{t} = \frac{4\pi n l}{\theta}.$$

Of the quantities involved in this relation, n and l are readily determined, and θ is equal to $\frac{1}{2}(a'ca)$. In practice it would be very difficult to measure $a'ca$ with any accuracy, and hence it is necessary to place a lens between a and m or between c and M , in such a position as to give *real* images of a , at a and a' . The distance aa' can then be measured, and θ evaluated in terms of the several distances involved. These distances are then easily measured, and V can be calculated with considerable accuracy.

If a long tube containing water or other transparent medium be placed between c and M , the displacement, from a to a' , will be greater or less, according as the velocity of

light in the given medium is less or greater than the velocity in air. Experiment shows that light travels more slowly through a dense than through a rare medium, that is, the greater the refractive index of the medium, the less is the velocity of light through it.

The velocity of light, as determined by Foucault, was about 298,000,000 metres per second.

Taking the mean of recent determinations, we get the velocity of light *in vacuo* to be 300,574,000 metres per second, or 186,700 miles per second.

CHAPTER X.

SIMPLE OPTICAL INSTRUMENTS AND EXPERIMENTS.

74. Artificial Horizon. The altitude* of a star is frequently determined by a method based on the laws of reflexion. The accuracy of the results obtained by this method furnish an indirect, but rigorous proof of the truth of these laws. A vertical divided circle, adjusted in a vertical plane, carries a telescope TT (Fig. 84) which can be rotated round an axis passing through the centre of the circle. In making an observation the telescope is first pointed to a particular star, and the reading on the circular scale, for this position of the telescope, is accurately noted. The telescope is then turned into the position T'T', so as to view the image of the star, formed by reflexion from the *horizontal surface* of mercury contained in the vessel M. The reading of the scale corresponding to this position is again noted, and the difference between the two readings, that is, the angle

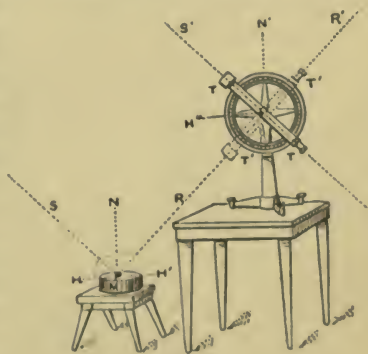


Fig. 84.

* The altitude of a star is the angle between the direction of the star and the horizontal.

$S'P'R$, gives twice the altitude of the star. For, *assuming the laws of reflexion to be true*, we have—

$$SPN = NPR,$$

and therefore—

$$SPH = RPH'.$$

but—

$$RP'H'' = RPH' \quad (\text{Euc. i. 29})$$

therefore—

$$RP'H'' = SPH$$

also—

$$S'P'H'' = S'P'H.*$$

Therefore we have—

$$S'P'R = S'P'H'' + H''P'R = 2SPH.$$

But SPH is the altitude of the star, therefore $S'P'R$ is twice the altitude of the star.

The accuracy of the results obtained by this method conclusively proves the truth of the laws of reflexion.

75. Hadley's Sextant. The sextant is an instrument employed for measuring the angle between two distant

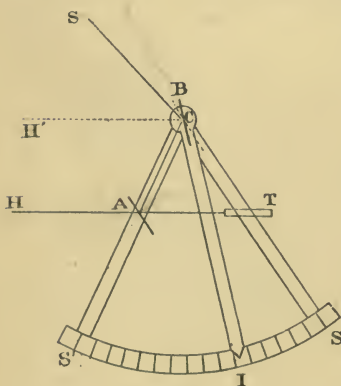


Fig. 85.

objects, as seen from the position occupied by the observer. The principle of its action has already been explained in Art. 27. The essential parts of the instrument are shown in Fig. 85. The frame is made up of the circular arc SS' and the two arms SC and $S'C$. These two arms, which are radii of the circle of which SS' is a segment, intersect at C the centre of the circle, and CI is an index arm which can be rotated

about an axis passing through C . Two plane mirrors, A and B , are attached to this arrangement; A is fixed on the arm $S'C$, and B is attached, at C , on the index arm

* SP and $S'P'$ are parallel. Compare footnote to Art. 72.

CI; both mirrors being perpendicular to the plane of the paper. The mirror A is unsilvered, or only partially silvered, so that an observer looking through the telescope T, which is directed towards A, can see objects in the direction TH. When I is at S, the planes of the mirrors, A and B, are parallel, so that any ray H'C, incident on B parallel to HT, is reflected along CA to A, and thence, along AT, to T.

The observer looking through T thus sees objects in the direction TH (or CH'), both directly through A, and by successive reflexion from B and A respectively.

On moving the index arm CI towards S', other objects, in addition to those seen directly through A, are brought into view, and if, when any particular object, in the direction CP, is brought into the field of view, the arm CI has been turned through an angle θ , then, by the principle of Art. 25, the angle PCH' is equal to 2θ . That is, the angle between an object seen in the direction CH', and another object in the direction CP, is equal to twice the angle SCI.

Hence, in determining the angle between any two given objects, the instrument is first adjusted until one of the objects is seen directly through A, and also by reflexions from B and A. The index I will then be at the zero of the scale on SS'. The arm CI is then moved until the other object, seen by reflexions from B and A, appears to coincide with the first object still seen directly through the unsilvered part of A. The required angle is then obtained by doubling the angle SCI which is given by the reading of the scale. Usually the scale is graduated on the principle of marking half-degrees as whole ones, so that the direct reading gives the required angle.

76. Images produced by a plate with parallel faces. Let O (Fig. 86) represent an object placed in front of the plate. Rays reach the nearest face of the plate in all directions from O. Consider the ray Oa. It is partially reflected from the first face at a, and an image due to this reflexion is seen at I. But a portion of the light

incident at a is refracted into the plate along ab , and, on incidence, at b , on the second face of the plate, a portion is reflected along bc , and the remainder refracted out into the air. The first portion, travelling along bc , again suffers partial reflexion and refraction at c , and the emergent ray, cf , gives rise to another image I' , fainter than the first at I , because of the loss of light at b and c . Similarly, after reflexion at d , and refraction at e , the light

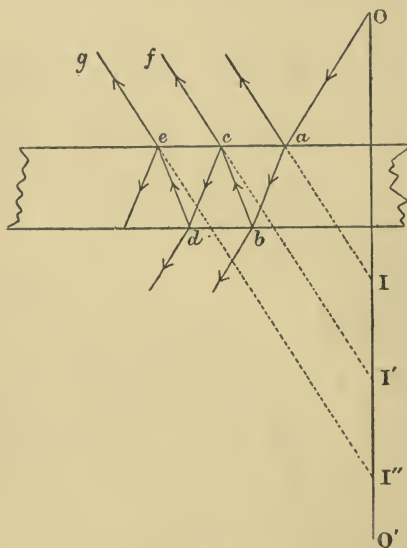


Fig. 86.

emergent along eg gives rise to another image I'' , fainter than that at I' . In this way, by continued reflexion and refraction, a series of images are formed on the line IO' ; each member of the series becoming fainter and fainter as the number of reflexions and refractions by which it is produced are increased.

These images can be plainly seen by holding a candle in

front of a looking-glass and viewing the image obliquely. A number of images, arranged along a normal to the mirror, will be seen; the first or nearest image, formed by reflexion at the first face, is rather faint; the next, formed by the first reflexion at the silvered surface, is very bright; the remainder are fainter, and decrease in intensity as their distance from the mirror increases. The number of images seen depends upon the polish of the reflecting surfaces, for, after a certain number of reflexions and refractions, the quantity of light reaching the eye becomes too small to excite the sensation of vision, and the loss of light by reflexion at any surface depends upon the degree of polish of that surface. In performing this experiment it will be noticed that the first image increases in intensity as the angle at which it is seen is increased. This shows that the quantity of light reflected from a glass surface increases as the angle of incidence increases.

77. Total reflexion prisms. Let ABC (Fig. 87) represent the section of a prism. The angle ABC is a right angle. Imagine light incident, along PQ , normal to the face AB . It will pass on undeviated until it is incident on the face AC at an angle of 45° . This angle being greater than the critical angle for glass and air, the light is totally reflected at AC , along QR , a normal to the face BC , and emerges from the prism without suffering further deviation. The light is thus turned through a right angle with little or no loss of light, and hence the arrangement is much better than a mirror placed in the position AC .

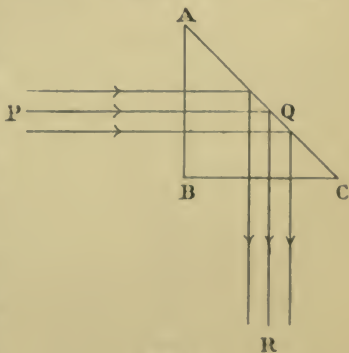


Fig. 87.

Wollaston's prism, sometimes called the *camera lucida*, is a totally reflecting prism with four angles, generally employed as an aid to sketching. A section of the prism is shown in Fig. 88; the angle ABC is a right angle, ADC is 135° , and the other two angles each $67\frac{1}{2}^\circ$. Light incident normally on BC , in the direction PQ , is totally reflected from the face DC to the face DA , whence it is totally reflected along RS normally to the face AB . To an eye

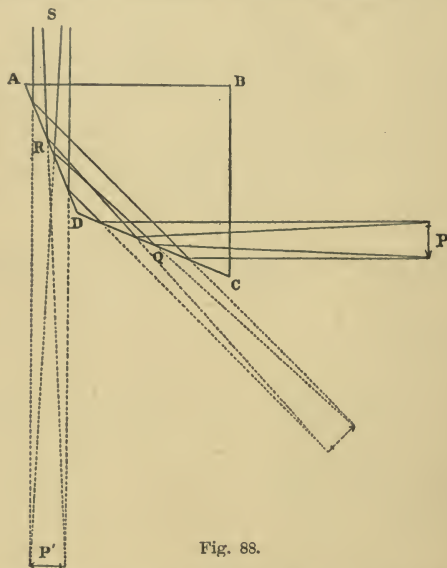


Fig. 88.

looking along SR , objects in the direction QP are seen in the direction SRP' , and the image thus seen may be traced on a sheet of paper placed at P' , vertically below S . The sheet of paper is seen past the edge A of the prism, while the image is seen by reflexion from the face AD . It is important that the image should be in the plane of the paper, for then paper, pencil, and image are seen with the same focussing of the eye. For this reason a concave lens of

short focal length is placed in front of the face BC when the object to be sketched is very distant. By adjusting the height of the prism in its stand (Fig. 89), the image can

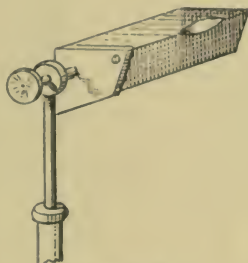


Fig. 89.

then be made to coincide with the plane of the paper. The two reflexions at the faces CD and DA are necessary to give an erect image.

78. The magnifying glass. A magnifying glass is simply a convex lens of short focal length employed to obtain magnified *virtual* images of small objects. The lens is placed at a distance less than its focal length from the object to be viewed, AB (Fig. 71), and, as explained in Art. 59, a virtual magnified image is formed at A'B', which, when the position of the lens is properly adjusted, can be clearly seen by an eye at E. A *simple microscope* is merely a magnifying glass mounted on a stand and provided with suitable accessories for the examination of small objects.

79. The camera obscura. The principle of this arrangement is indicated in Fig. 90. At the top of a small tent or wooden structure is a small cylindrical or cubical box, which contains a mirror R and a lens L, arranged as shown in the figure.

The mirror is inclined at 45° to the horizontal, and reflects the rays coming from any external object AB on

to the lens L , which forms an image $A'B'$ on a white table or screen placed vertically below it. The room is

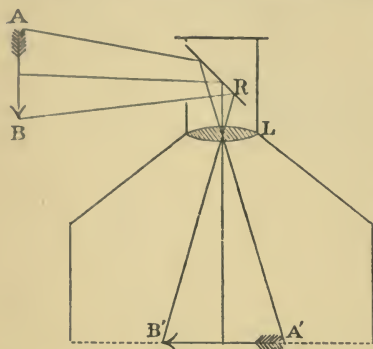


Fig. 90.

perfectly dark, and the inside carefully blackened, so that the image thus cast upon the screen or table may be clearly seen. The box containing the mirror and lens can be rotated, and thus images of all objects surrounding the tent are in turn cast upon the table. Instead of the mirror and lens it is better to employ a totally reflecting prism with the $\frac{1}{2}$ faces,

which are turned towards AB and $A'B'$ respectively, convex and concave. The curvature of the concave surface is less than that of the convex, and the arrangement thus acts as a convex lens and a mirror combined.

The **photographic camera** is constructed on the same principle. If a lens be substituted for the aperture in the pinhole camera described in Art. 4, we have the essentials of a photographic camera. Images of external objects are formed by the lens on a screen at the back of the camera; and, if a suitably prepared photographic plate be substituted for the screen, the action of the light on this plate is such that when subjected to proper chemical treatment a *negative* is obtained from which the ordinary photographs can be printed on sensitized paper.

80. The magic lantern. The principle and construction of this instrument will be understood from a study of Fig. 91. A source of light (lamp, lime-light, or electric light) is placed at S , at the focus of the concave mirror M . A beam of parallel light is thus reflected from M on to the lens L , which concentrates it on the slide s , which

contains, in an inverted position, a transparent photograph, or other suitably prepared representation of the picture to be shown on the screen. The lens O then forms on the screen, R , a magnified inverted image of the picture in the slide. The latter being itself inverted, the image on the screen is erect, and magnified in the ratio $RO:Os$. The tube carrying the lens O can be slid backwards and forwards, so that a clearly defined image can always be focussed on the screen. The magnification possible with any instrument is limited by the intensity of the light illuminating s . For the quantity of light falling on s is also the quantity which illuminates the image on the screen, and hence the intensity of this image diminishes as its area is increased.

81. Experimental illustration of the deviation produced by a prism. In Art. 52 the phenomenon of deviation of light by a prism was illustrated by the displacement of the *virtual* image of a slit seen through the prism. By application of the principle of Art. 65 we can, however, illustrate deviation by noticing the displacement of a *real* image, obtained by means of a lens and prism. The apparatus described below may be employed for this purpose, and will also serve to give rough measurements of the deviation produced.

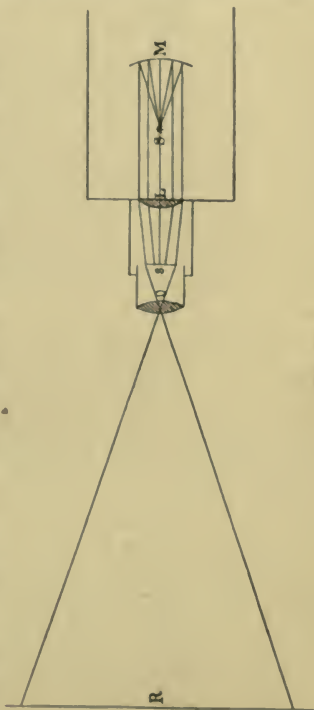


Fig. 91.

A small truly circular table, about 30 cm. in diameter, is fitted round its edge with a strip of thin tough paper, in such a way that the upper edge of the strip projects about 5 cm. above the face of the table. On this projecting edge, about half-way up, a scale, showing degrees, is marked all

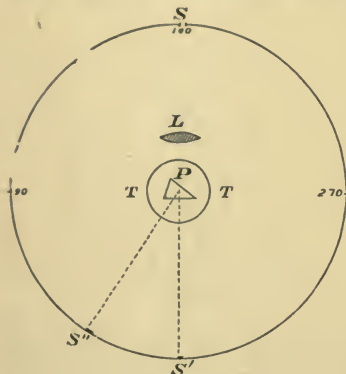


Fig. 92.

round the strip. At the 180th division on the scale, a narrow vertical slit, *S* (Fig. 92), about 2 cm. long, is so cut in the paper that one edge accurately coincides with this division. The apparatus being placed in a dark room, the slit is illuminated by a properly shaded sodium flame, and a suitably mounted convex lens, *L*, is fixed on the table, between the slit and the centre, in such a

position that a clearly defined image of the slit, having one edge coincident with the zero on the scale, is obtained on the paper strip at *S'*. At the centre of the table is fixed a small stand, *TT'*, which can be rotated round a central vertical axis. On this stand a prism can be placed with its edge vertical, and fixed so that the plane bisecting its refracting angle passes through the axis of rotation of the stand. On placing the prism in position, and rotating the stand until the rays of light coming from the slit, through the lens, are refracted through the prism, it will be seen that the position of the image of the slit is changed, and that the change of position indicates that the rays are deviated by the prism in a direction away from its refracting edge. As the prism is rotated the position of the image changes, indicating that the magnitude of the deviation produced depends upon the position of the prism relative to the incident light. If, for any position of the prism, the image

is formed at S'' , then the magnitude of the deviation is measured by the angle $S'PS''$, which may be at once read off on the scale. If the prism be rotated so as to cause the deviation to diminish, it will be found that, as the prism is rotated, always in the same direction, the image will travel at a gradually decreasing rate towards S' , and at a certain point will become stationary, and then turn back in the opposite direction. The deviation at the instant at which the image is stationary is the *minimum deviation*, which can thus be determined by noting, on the scale, the division at which the image ceases to advance towards S' and begins to turn back. The image obtained on the scale, after the interposition of the prism, is not clearly defined except at, and near, the position of minimum deviation, and consequently measurements made near this position will be more correct than for other positions. Accurate measurements of deviation are made by means of the *spectrometer*.

82. Determination of the focal length of a lens. The experimental determination of focal length is of great importance. The methods adopted depend upon the nature of the lens, and upon the degree of accuracy required. We shall consider a few of the simpler approximate methods for each class of lenses.

I.—Convex lenses.

1. The simplest method of determining the focal length of a convex lens is to allow a beam of parallel light to be incident on the lens, in a direction parallel to the principal axis, and then to measure the distance of the focus of the refracted pencil from the lens. For this purpose the lens is mounted in a suitable stand or clip, with its axis parallel to a graduated bar of wood, along which the stand slides. At one end of this bar, and at right angles to its length, a thin paper or ground-glass screen is fixed, with its centre approximately on the same level as the principal axis of the lens. This arrangement, with the screen towards the observer, is then pointed towards the sun or some other well-

defined object, and the position of the lens adjusted until a clearly defined image of the sun, or other object chosen, is formed on the screen. The distance between the lens and the screen, as indicated by the gradations on the bar, gives the required focal length, for if the object be sufficiently distant the image is approximately at the principal focus of the lens.

2. This method is an application of the formula—

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}.$$

The lens is mounted in one of the uprights of the optical bench, with its axis parallel to the length of the bench. In two other uprights, placed one on each side of the lens, are fixed a lighted candle and a screen, the flame and the centre of the screen being adjusted to the level of the principal axis of the lens. By properly adjusting the positions of these two uprights, relative to that carrying the lens, a sharply defined image* of the flame is formed on the screen. The distances v and u are then determined by noting on the scale of the bench the distances between the screen and the lens and between the candle flame and the lens. The value of f is then calculated from the relation—

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}.$$

These measurements should be made for several different values of v and u , and the mean of the results taken as the mean value of f .

Instead of the candle flame it is better to employ a small sharply defined object brightly illuminated by a suitably placed light. Two fine wires stretched across a hole in a piece of cardboard, a piece of fine wire gauze, or a piece of glass with a scale etched on it answer the purpose extremely well.

* See Art. 68.

3. **The displacement method.** Let A and B (Fig. 93) represent respectively the positions of a bright object and a screen. Then, if a *magnified* image of the object A be formed on the screen at B by a lens placed at C, a *diminished* image can also be obtained on the screen by placing

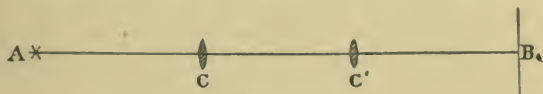


Fig. 93.

the lens at a point C' such that $BC' = AC$, for, if AC and BC are conjugate focal distances, then the *equal* distances AC' and BC' are also conjugate. Let AB be denoted by l and CC' by a , then if AC and C'B be each denoted by p , and CB and AC' be each denoted by q , we have—

$$AB = AC + CB$$

$$\therefore l = p + q \quad (1)$$

and—

$$CC' = AC' - AC$$

$$\therefore a = q - p. \quad (2)$$

Therefore from (1) and (2) we get—

$$p = \frac{l - a}{2},$$

and—

$$q = \frac{l + a}{2};$$

but by usual formula,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f};$$

and here—

$$u = p \text{ and } v = -q.$$

$$\therefore -\frac{2}{l + a} - \frac{2}{l - a} = \frac{1}{f}.$$

$$\therefore -\frac{4l}{l^2 - a^2} = \frac{1}{f}.$$

$$\therefore f = -\frac{l^2 - a^2}{4l}. \quad (3)$$

Hence, by measuring l and a , f may, by application of this formula, be readily determined.

The measurements of l and a are made by means of the optical bench in a method similar to that explained above.

A particular case of this method is applied in Silbermann's Focometer (Fig. 94).

When a , in formula (3) above, becomes zero, we have, neglecting sign—

$$f = \frac{l^2}{4l} = \frac{l}{4}.$$

When this is the case the points C and C' in Fig. 93 are evidently coincident, and we have $AC = CB$. That is, image and object are equidistant from the lens, and are therefore equal in size (Art. 60). This is the fact

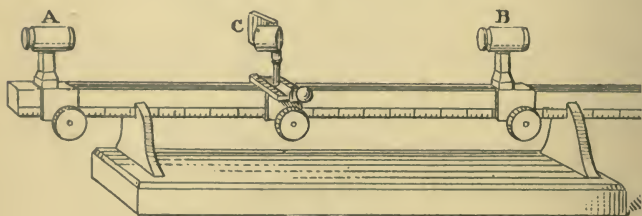


Fig. 94.

made use of in this instrument, which consists of a fixed scale carrying three slides, A, B and C. Mounted in A and B are two thin pieces of horn or other semi-transparent substance having exactly similar and equal patterns ruled on them. The slide C carries the lens whose focal length is to be determined. The positions of these slides must be adjusted until the image of the pattern in A is seen to coincide exactly with that in B. It will then be found that A and B are equidistant from C, and the distance between A and B, read off on the scale, gives l , from which f ($= \frac{l}{4}$) is easily calculated.

II.—Concave lenses.

We have already seen that with a concave lens the image is always virtual, and cannot therefore be received upon a screen. This renders it difficult to determine the focal length of a concave lens, but the following methods may be adopted with fairly accurate results:—

1. One face of the concave lens is covered with a circular piece of black paper, through which two large pinholes have been made, on a diameter of the circle, at points equidistant from the centre. A beam of parallel light is then directed on the lens in a direction parallel to the principal axis. All the incident rays except those passing through the pinholes are stopped by the black paper, and if a screen be placed behind the lens, two bright spots will be formed upon it at the points where the rays passing through the pinholes meet its surface.

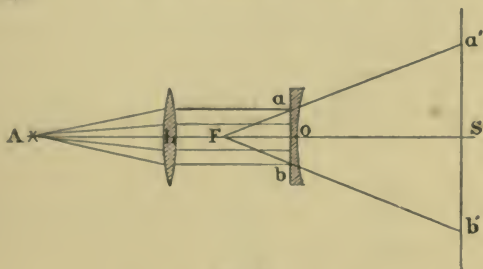


Fig. 95.

Let a and b (Fig. 95) represent the positions of the pinholes, then, the incident light being parallel, the rays refracted through at a and b diverge from the focus F , and bright spots of light are formed, at a' and b' , on a screen placed at any point S , behind the lens.

From the figure we have—

$$\frac{a b}{a' b'} = \frac{F O}{F S} = \frac{F O}{F O + O S}.$$

Therefore, if the focal length of the lens be denoted by f , we get—

$$\frac{a b}{a' b'} = \frac{f}{f + O S}.$$

From this relation f can be determined when ab , $a'b'$ and OS are known. The distance OS is read off on the scale of the optical bench on which the screen and lens are placed, and the lengths ab and $a'b'$ are measured with a pair of compasses and a fine scale. This experiment should be conducted in a darkened room, and the beam of parallel light obtained by placing a source of light, A , at the focus of a convex lens, L , fixed in an upright behind the concave lens. Only very rough results can be obtained by this method.

2. Three uprights carrying respectively a bright object, a *convex* lens and a screen, are arranged on an optical bench in such positions that a clearly defined image of the object is formed on the screen. The position of the screen being noted, the concave lens whose focal length is to be determined is mounted in an upright previously placed between the convex lens and the screen. By properly adjusting the position of the concave lens and the screen an image of the object can be again obtained, and by noting this second position of the screen and that of the convex lens data are obtained from which the required focal length may be determined. Let P (Fig. 96) denote the position of the

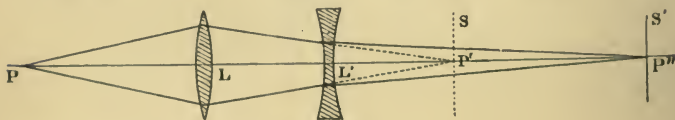


Fig. 96.

object, and P' the position of the image formed on the screen at S by the convex lens L . If the *concave* lens be placed at L' in such a position that $L'P'$ is less than its focal length, then the rays converging to P' are rendered less convergent, and made to converge to a more distant point P'' . If the screen be placed at S' an image of the object at P is formed on it; this image may be considered as the image of that at P' , and if $L'P'$ and $L'P''$ be measured the focal length of the concave lens may be calculated from the relation—

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f},$$

where $L'P'$ is denoted by u , $L'P''$ by v , and the required focal length by f .

3. It can be shown* that if two thin lenses of focal lengths, f_1 and f_2 , be placed in contact so as to act as one compound lens, then the focal length, F , of the combination is given by the relation—

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}.$$

If a concave lens of focal length, f_1 , be combined in this way with a convex lens of *shorter* focal length, f_2 , the combination is evidently equivalent to a convex lens, and its focal length, F , may be determined by either of the methods given above. Similarly f_2 may be determined, and the required focal length f_1 may then be calculated from the relation just given.

* See "Examples in Heat and Light," Ex. V. 6.

EXAMINATION QUESTIONS.

QUESTIONS SET AT LONDON UNIVERSITY EXAMINATIONS.

Matriculation.

1. What is meant by the Focal Length of a Convex Lens? Show how to find it (1) by aid of the sun, (2) by an artificial flame.

June, 1874.

2. An arrow, pointing towards the observer, is seen by internal reflexion in an isosceles right-angled prism. Explain the difference in, and give a sketch of, the images seen, according as the prism is three-sided, or a four-sided Wollaston prism.

Jan., 1879.

3. What is the focal length of a lens, and how would you determine it experimentally? In the case of a convex lens, if the object be as near as possible to the image, where must the lens be?

Jan., 1880.

4. Show by a drawing how you would employ a right-angled isosceles glass prism to bend a beam of light at right angles. Will any light be lost at the hypotenuse? State fully the reasons for your answer.

Explain the formation of an image by a convex mirror.

Jan., 1881.

5. How would you determine the focal length of a convex lens if sunlight were not available?

Jan., 1882.

7. What is the focal length of a lens? How may the focal length of a concave lens be determined?

Jan., 1884.

8. A candle is placed at a fixed distance opposite a wall. A convex lens, held between the candle and the wall, throws on the wall a well-defined magnified image of the candle flame when it is 1 foot from the candle, and a well-defined diminished image when it is 11 feet from the candle. Find the focal length of the lens.

Jan., 1885.

9. How is the focal length of a convex lens best determined without the aid of sunlight?

An object is placed 8 inches from the centre of a convex lens, and its image is found 24 inches from the centre on the other side of the lens. If the object were placed 4 inches from the centre of the lens, where would the image be?

June, 1886.

APPENDIX.

QUESTIONS SET AT LONDON UNIVERSITY MATRICU-
LATION EXAMINATIONS UNDER THE NEW (1888)
REGULATIONS.

Wednesday, June 20, 1888—Afternoon, 2 to 5.

HEAT AND LIGHT.

Examiners { R. T. GLAZEBROOK, Esq., M.A., F.R.S.
Prof. A. W. REINOLD, M.A., F.R.S.

1. If 3,000 cubic inches of air at 0° C. expand by 11 cubic inches for each degree rise of temperature, find the volume at 100° of a quantity of air which at 50° measures 100 cubic inches, the pressure being supposed to undergo no change.

2. Define *specific heat* and *capacity for heat*, and explain how you would determine the specific heat of oil.

3. Explain the statement that the latent heat of water is 80. To a pound of ice at 0° are communicated 100 units of heat (pound-degrees Centigrade). What change of temperature does the ice undergo, and in what way is its volume altered?

4. Describe an experiment to illustrate the convection of heat, and trace the processes by which heat is conveyed through a hot-water heating apparatus from the boiler-fire to the walls of a room heated by the pipes.

5. Describe and explain some method of determining the velocity of light.

6. How would you use Rumford's (shadow) photometer for comparing the illuminating powers of two sources of light? State clearly the principle of the method.

7. A bright object, 4 inches high, is placed on the principal axis of a concave spherical mirror, at a distance of 15 inches from the mirror. Determine the position and size of its image, the focal length of the mirror being 6 inches.

8. What is a *spectrum*? Describe carefully what you would see in looking through a glass prism at a point of light in a dark room. Illustrate your answer with a figure.

Wednesday, January 16th, 1889.—Afternoon, 2 to 5.

HEAT AND LIGHT.

Examiners { Prof. G. F. FITZGERALD, M.A., F.R.S.
R. T. GLAZEBROOK, Esq., M.A., F.R.S.

1. Explain why in reading a barometer it is necessary to correct the reading for the temperature of the mercury. A barometer with a glass scale reads 755 mm. at 18°C .; find the reading at 0°C . The apparent coefficient of expansion of mercury in glass is $\cdot 000155$, and the coefficient of linear expansion of glass is $\cdot 0000089$.

2. What is the law connecting the change of pressure of a gas at constant volume with the change in its temperature as measured by a mercury thermometer? Describe experiments to verify this.

3. How would you distinguish between *vaporisation* and *ebullition*? Does the boiling point of a liquid depend on the pressure on its surface? Illustrate your answer with an experiment.

4. Calculate the temperatures Centigrade corresponding to 100°F .— 40°F . 0°F . 98°F .

5. If a small hole be made in the shutter of a darkened room an inverted image of objects outside is formed on a screen placed within the room. Explain this.

6. Distinguish between a real and a virtual image formed by optical means. A candle is placed in front of a piece of flat glass, and on looking into the glass an image of the candle is seen; show how to determine the position of this image. Is it real or virtual?

7. A person looks at an object through a concave lens of 1 foot focal length, the object being 5 feet beyond the lens. Draw a figure showing the paths of the rays by which he sees the image formed, and determine its position.

8. Describe how to produce a pure spectrum.

Wednesday, June 12th, 1889—Afternoon, 2 to 5.

HEAT AND LIGHT.

Examiners { R. T. GLAZEBROOK, Esq., M.A., F.R.S.
Prof. G. F. FITZGERALD, M.A., F.R.S.

1. Describe an experiment for showing that the volume of a gas at constant pressure increases by approximately $\frac{1}{273}$ rd of its volume at 0° C. for each rise of 1° C. of temperature. The volume of a mass of gas at a pressure of half an atmosphere and temperature 15° C. is 150 c.c.: find the volume when the temperature is 303° C. and the pressure one atmosphere.

2. Distinguish between *capacity for heat* and *specific heat*; and describe some method of measuring the specific heat of a metal.

3. Distinguish between a *vapour* and a *gas*. What is meant by the *dew-point*, and how may it be found?

4. Describe an experiment by which you would show that ice contracts when it melts, and that the resultant water goes on contracting if it be warmed.

5. Explain how observations on Jupiter's satellites lead to a determination of the velocity of light. What data are necessary in order to make the calculation?

6. A lens of focal length of one foot is placed at distances (*a*) of three feet, and (*b*) of six inches, from an object two inches long. Draw figures showing the paths of rays of light from each extremity of the object, and find the position and size of the images formed in the two cases respectively.

7. Explain clearly why a narrow slit, a prism, and at least one lens are required in order to form a pure spectrum on a screen.

8. A circular uniform source of light, two inches in diameter, is placed at a distance of ten feet from a sphere two inches in diameter. Calculate, approximately, the diameters of the umbra and penumbra cast on a screen five feet beyond the sphere.

MATRICULATION EXAMINATION.

Wednesday, January 15th, 1890.—Afternoon, 2 to 5.

HEAT AND LIGHT.

Examiners { Prof. G. T. FITZGERALD, M.A., F.R.S.
R. T. GLAZEBROOK, Esq., M.A., F.R.S.

1. Describe how to measure the apparent coefficient of expansion of a liquid by means of a weight thermometer.

2. Define specific heat. How would you determine the specific heat of alcohol?

3. Describe how maximum and minimum thermometers are constructed, and how they are to be used.

4. Describe how the heating of buildings, (a) by hot water-pipes, (b) by steam, depends on convection and conduction of heat, specific heat, and latent heat.

5. Describe how to measure the relative intensities of two sources of light by the shadow photometer.

6. A concave mirror of 2 feet focal length is placed 1 foot from an object; find the change in the position of the image produced by moving the object 1 inch nearer the mirror.

7. Draw a diagram explaining the formation of an image of a given object by a concave lens. Can such an image be made larger than the object? Give reasons for your answer.

8. How would you show, experimentally, that a ray of light is deviated by passing through a prism, and how would you measure the deviation?

ANSWERS.

HEAT.

EXAMPLES I.

- 4.—(1) -94° F. ; -56° R.
(2) 24.4° C. ; 19.5° R.
(3) -30° C. ; -22° F.
(4) 32° F. ; 0° R.
(5) 10° C. ; 8° R.
(6) 20° C. ; 16° R.
(7) 80° C. ; 176° F.
(8) 197.6° F. ; 73.6° R.
(9) -10° C. ; -8° R.
(10) 62.5° C. ; 144.5° F.

EXAMPLES II.

1. 1.000089.
2. 1.
3. 1.0007 ; 999961.
4. 999975 ; 1.0013.
5. 1.00012 ; 1.00032.
6. 100.084 ; 99.984.

EXAMPLES III.

8. .03 cm.
9. 72.2 cm.

EXAMPLES IV.

4. $\frac{d_{100}}{d_{-100}} = .9832.$
 6. 10.1476 grams.
 7. .00003225.

EXAMPLES V.

11. .000017.
 16. 100° C.
 17. .000029.
 21. $\frac{1 + \delta}{1 + \Delta}.$

EXAMPLES VI.

8. 0.003821.
 11. 663.77.
 12. (i) P, T', $\frac{T' V}{T}.$
 (ii) $\frac{T' P}{T}, T, V.$
 13. $\frac{1}{c_r} = 273.68.$
 17. .0841 gram.
 18. 1.00428 (nearly).
 19. 313° C.
 20. 273° C.
 22. -459.4° C.

EXAMINATION QUESTIONS (page 72).

2. 534.16 c.cm.
 3. 450.531 cub. in.
 12. 64.1° C.
 15. .185 cub. ft. (nearly).
 16. 1.54 c.cm.
 17. 345.2 cub. in.; 577.4 cub. in.

EXAMPLES VII.

9. 25.89° ; 9.12 grams.
10. 13.1579 grams.
12. $.0903$ (nearly).
13. $.6153$.
15. 745.3° .
16. 77.29° (nearly).
17. 1.005 ; 1.027 ; 1.016 .

EXAMPLES VIII.

4. 95.58 pound-degrees.
7. 79.561 .
8. $.1108$.
9. $.0334$.
10. $.9167$.

EXAMPLES IX.

11. 966.6 ; 144 .
13. 72.75 grams.
15. 65.3° C.
16. 32.26 grams. (Copper to be taken at 1000° C.)

EXAMINATION QUESTIONS (page 130).

10. 5.66 lbs.
13. 12.5° (nearly).
15. 44.4 gram-degrees. (Ex. VIII. 2.)
16. 495.3 .
17. 25.2 ; 1.851° C.
18. $.0329$.
19. 31.85° C.
20. $.932$.
21. 79.705 .
23. 28.923° C.
24. 80 .
26. 47.5 gram-degrees; 59.375 grams.

EXAMPLES X.

4. 1.205 grams.
5. -5.2°C .
6. 10.1°C . ; 77.1 per cent.
7. $h = 72.76$ per cent.

EXAMPLES XI.

5. .0384.
6. 12,000,000 gram-degrees.
7. 125.28 gram-degrees.
9. .00013.

LIGHT.

EXAMPLES I.

7. 100 cm. ; 20 cm.
8. Diameter of umbra = 5.856 cm.
Diameter of penumbra = 9.293 cm.
9. .00894 sq. cm. (nearly) ; $A'B' = 13.3$ cm.
10. 3 : 4.
11. $I_D : I_B : I_C :: 3\sqrt{3} : 8 : 8$.
12. $a^2 : b^2$.
13. $(115)^2 : (201)^2$.
14. 80 cm. from the less intense light.
15. (a) Screen between the lamps, $2\frac{2}{3}$ ft. from 16-power lamp.
(b) Screen outside lamps, 24 ft. beyond 16-power lamp.

EXAMPLES II.

4. 12 in., 24 in., 36 in., etc. ; 12 in. ; 12 in.
5. 60° .
6. 3 ft.
7. 100° .

EXAMPLES III.

7. (1) Distance < 12 ; (2) distance < 12 .
 8. 13.5 in.
 9. $\frac{1}{O} = \frac{f}{u-f} = \frac{f}{3f-f} = \frac{1}{2}$.
 10. 12 in.
 11. .025 in.; 9 in. behind.
 13. $3\frac{3}{4}$ in. behind; $\frac{3}{4}$.
 17. $\frac{1}{2}$, virtual, $\frac{f}{2}$ behind.

EXAMINATION QUESTIONS (page 55).

9. Real and one-third as large as object; 1 ft. from the mirror; inverted.
 16. 6 in.
 19. 1 ft. from the mirror; inverted; three times as large; at the centre of curvature.
 21. Object 3 in. in front of mirror; image $1\frac{1}{2}$ in. behind mirror.
 26. Between candle and gas flame, 2 ft. from former and 6 ft. from latter; or, on the line passing through the lights, 4 ft. from the candle and 12 ft. from the gas flame.

EXAMPLES IV.

5. $1\frac{1}{2}$ nearer.
 6. .654 (nearly).
 7. $\frac{9}{7}$.
 10. $\frac{2\sqrt{3}}{3}$.
 11. 4.38 in.
 12. 225,563,909.8 metres per second.

EXAMPLES V.

9. $.625^\circ$.
 10. $\sqrt{2}$.
 15. $3 \pm \sqrt{3}$.
 16. Real; 5 in.

17. If image is real, $f = 8$ cm.
 " " " virtual, $f = 13\frac{1}{3}$ cm.
 18. Virtual, $\frac{1}{7}$ object; $4\frac{2}{7}$.
 19. 9 in.
 23. 4 in.
 25. $4\frac{2}{11}$ in. in front of plane mirror.
 26. 15 cm. from mirror.

EXAMINATION QUESTIONS (page 101).

3. 25 : 1.
 5. 5 in.
 6. 5 in. from the bright point.
 8. The images formed are 6 and $8\frac{1}{4}$ in. below the surface of the water.
 11. $\frac{3}{4}$.
 17. 18 in. (To obtain a *real* image with a *convex* lens.)
 20. $\frac{9}{8}$.
 23. First image $8\frac{1}{4}$ in. from first lens; diameter $\frac{3}{4}$ in. Second image $15\frac{1}{4}$ in. from second lens; diameter $1\frac{1}{4}$ in.
 26. 4 in.
 28. See Examples V. 5.
 31. 1.48.
 33. 2 ft. from lens on the same side as the object; 6 in. high.

EXAMINATION QUESTIONS (page 138).

8. 11 in.
 9. On the same side as the object, and 12 in. from the centre.

EXAMINATION PAPERS: APPENDIX.

June, 1888.

1. $V_{100} = 115.5$ cub. in.
 2. Arts. 38, 39, 40.
 3. Arts. 49, 50; 20° C.; Art. 52 (2).
 4. Arts. 81, 93.
 5. Arts. 72, 73.
 6. Art. 12.
 7. Between object and mirror, at a distance of 10 in. from the mirror; $2\frac{2}{3}$ in.
 8. Arts. 63, 65.

January, 1889.

1. 752·652 mm.
2. Art. 32 (iii).
3. Arts. 54, 60, 62, 63.
4. $37\frac{1}{2}^{\circ}$ C.; -40° C.; $-17\frac{1}{2}^{\circ}$ C.; $36\frac{3}{4}^{\circ}$ C.
5. Art. 4.
6. Arts. 18, 19, 21.
7. 10 in. from lens.
8. Art. 65.

June, 1889.

1. Art. 32 (ii); 150 c.cm.
2. Arts. 38, 39, 40.
3. Arts. 55, 76.
4. Arts. 29, 52 (2).
5. Art. 72.
6. $\left\{ \begin{array}{l} (a) \ 1\frac{1}{2} \text{ ft. on the other side of the lens; image } \frac{1}{2} \text{ size of the} \\ \text{object.} \\ (b) \ 1 \text{ ft. from the lens on the same side; image twice size} \\ \text{of the object.} \end{array} \right.$
7. Art. 65.
8. Umbra = 2 in.; penumbra = 4 in. (Ex. I. 2.)

January, 1890.

1. Art. 27 (1).
2. Arts. 39, 40.
3. Art. 11.
4. Art. 93.
5. Art. 12.
6. $\frac{1}{3}$ in.
7. Art. 58 (Fig. 69); Art. 57.
8. Art. 81.

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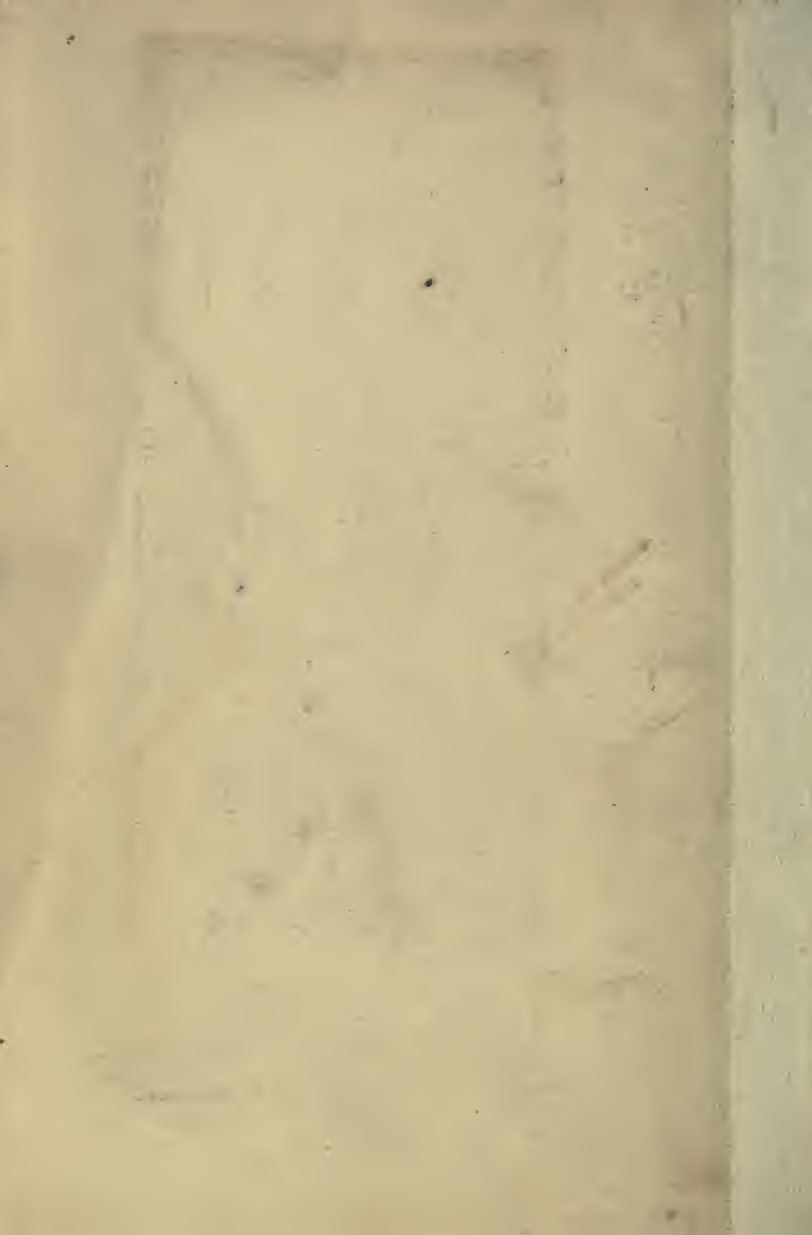
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